MENSURATION AND ELEMENTARY SURVEYING

BY

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"SURVEYING AND DELAYING"

EXAMINER SURVEY EDUCATION ADVISORY BOARD, OVERSEER EXAMINATION BOARD, PLEADERS' SURVEY EXAMINATION BOARD, ETC. ETC.

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CHAPTER I

INTRODUCTORY

1. Mensuration is that branch of Applied Geometry which is concerned with finding the length of lines, area of surfaces, and volume of solids.

2. British Linear measures are:

12 inches = 1 foot
3 feet = 1 yard*

Contracted notations are in. or ″ for inches, ft. or ′ for feet, and y. or yd. for yards, e.g.

58 yd. 2 ft. 7 in. or 58 yd. 2′ 7″
$5\frac{1}{2}$ yards = 1 pole or perch or rod
40 poles = 1 furlong
8 furlongs = 1 mile†

∴ One mile is a length of 1760 yd. or 5280 ft.
4 poles = 1 chain (Gunter's)‡

∴ 1 Gunter's chain is 22 yd. or 66 ft. long
and 10 Gunter's chains make one furlong
80 Gunter's chains make one mile.

One hundredth part of a Gunter's chain is called a link.

A chain is the length of a cricket pitch.

* The term inch is derived from the Latin *Uncia*, a thumb-joint. The standard of foot was taken from the span of the foot of a stalwart Welsh king. All these measures were of varying lengths, and by an Act of Parliament in 1855 the Imperial yard was defined as the distance between two gold plugs set in a bronze bar of a temperature of 62 degrees Fahrenheit; the bar is being preserved in the office of the Exchequer.

† Mile—Latin, *mille passus*, 1000 paces. A pace was a double step and, for a soldier, roughly $1\frac{3}{4}$ yd. So 1000 paces can be calculated as approximately $1000 \times 1\frac{3}{4}$ yd. = 1750 yd.

Furlong—In olden days fields were long, and furrows were $\frac{1}{8}$ mile long. Hence $\frac{1}{8}$ mile is called furrow-length or furlong.

‡ This chain was invented by Edmund Gunter (1581-1626) who was Professor of Astronomy at Gresham College, London.

3. The metric system :

In this system the unit is the metre, which was originally intended to be $\frac{1}{10000000}$th part of the distance from the Pole to the Equator of the earth. A bar supposed to represent this length and made of eridio-platinum, is kept at the *Palais des Archives*, in Paris. The latest determination of the length of this standard metre (1896) makes it 39·370113 in., but the last two figures are quite uncertain.

The multiples of a metre are denoted by using a Greek prefix and the sub-multiples, or the decimal fractions of a metre, are denoted by a Latin prefix.

SUB-MULTIPLES :

$\frac{1}{10}$ of a metre (m.) = 1 decimetre (dm.)

$\frac{1}{10}$ of a decimetre or $\frac{1}{100}$ of a metre = 1 centimetre (cm.)

$\frac{1}{10}$ of a centimetre or $\frac{1}{1000}$ of a metre = 1 millimetre (mm.)

MULTIPLES :

10 metres = 1 dekametre (Dm.)

10 dekametres or 100 metres = 1 hectometre (Hm.)

10 hectometres or 100 dekametres or 1000 metres = 1 kilometre (Km.)

Please note that the contracted notations in sub-multiples are in small letters and those of multiples, in capital letters.

For ordinary purposes a metre is taken to be a length of 39·37 inches. The breadth of the nail of the little finger on an average hand is about 1 cm. and the length of the nail-joint of that finger is about an inch.

It should be noted that dekametres and hectometres are rarely used.

1 Km. is nearly $\frac{5}{8}$ mile*

The metric system lends itself very readily to working with decimals; that is the great advantage of it.

* A historical sketch of the metric system: The want of a uniform system of weights and measures in the various countries of Europe has always been a serious hindrance to commerce. Attempts at uniformity were made as early as the time of Charles the Great (A.D. 800) but met with no permanent success. At various times in the past the term "pound" has been used for several hundred different weights and the word "foot" for at least two hundred different lengths. The French nation at the time of the French Revolution set themselves definitely to develop a system of scientific measurement. On May 8th, 1790, the National Assembly of France requested

4. Linear measures used in India are different in different parts of the country. But everywhere the unit is the *hath*, or cubit, which is a length of 18 inches of the British measure.

TABLE :

18 in.	= 1 hath or cubit
2 haths	= 1 yard
4 haths or 16 chataks	= 1 katha
20 kathas	= 1 rassi or bigha*
∴ 44 rassis	= 1 mile
One chatak	= 9 in.

Louis XVI to concert with the King of England, so that a society of scientific men from the two countries might determine at latitude N 45° the length of the pendulum beating the second, in order that this length might be adopted as the linear measure from which all the other standards should be derived. Political events rendered the co-operation of England impossible, but the French Commission was appointed, and on March 19th, 1791, presented their report, which was almost immediately adopted. They recommended that the standard of length should be the 10000000*th* (ten millionth) part of the distance from the earth's Equator to the Pole, and that the other standards should be derived from this. Two French mathematicians, Delambre and Méchain, measured the distance from Dunkirk to Barcelona and a "Commission of weights and measures," composed of twenty-two members selected from various countries of Europe, then undertook the final work based on the calculations of Delambre and Méchain, and deduced the length of the new unit, the metre, and the sizes of the multiples and sub-multiples of metres. The metric system was made compulsory in France in 1802, and such are its advantages that, despite the international jealousy, it has spread all over Europe. Russia and England are the last of the civilised countries to fall into line with the rest. The Weights and Measures Act of England in 1897 legalised the use of metric system in trade, and abolished the penalty for using or having in one's possession a weight or measure of the system. The Russian Government, in 1898, announced its intention of adopting. The later determinations show that the measurements of the earth made in 1791 were not quite exact, and the distance between the Pole and Equator is not really 10000000 metres but about 10000856 metres. Thus the metre is also an arbitrary standard, like the yard and other standards.

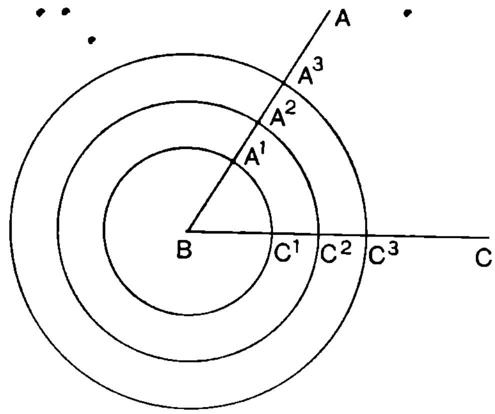
* This table gives the measures for a standard *bigha* as adopted for the Settlement operations of Bengal.

Chapter II

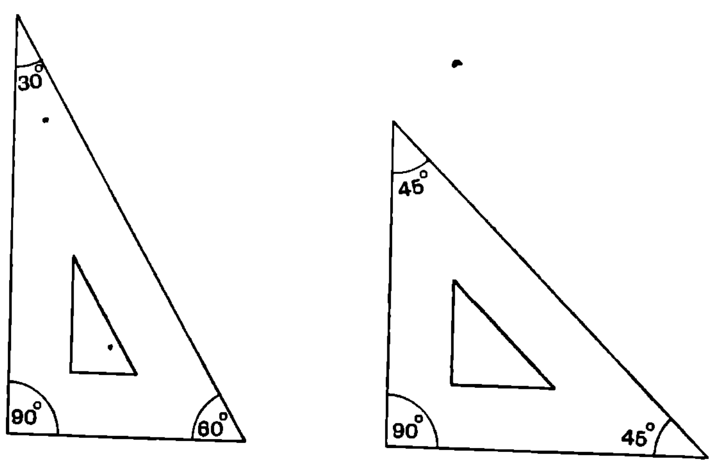
USE OF DRAWING INSTRUMENTS

5. In Geometry an angle is defined as the inclination of two straight lines.

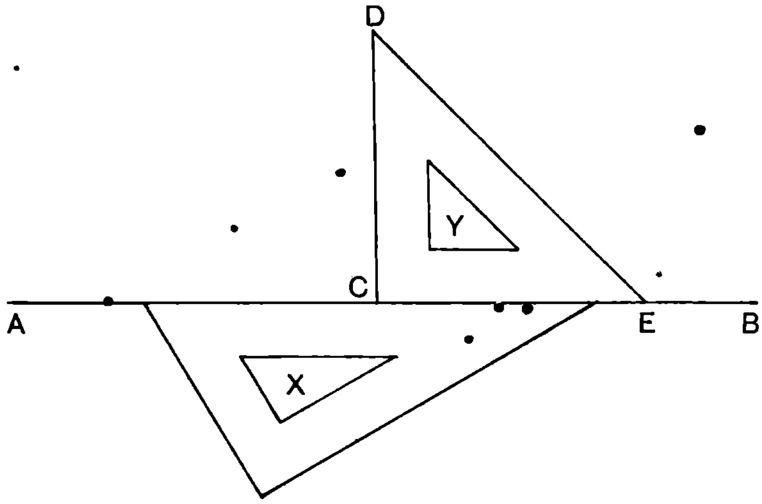
With B as centre and with different radii draw the various circles as in the diagram. It will now be seen that the arcs $A^1 C^1$, $A^2 C^2$ and $A^3 C^3$ etc., bear a constant ratio to the circumferences of the respective circles. Advantage of this truth is taken to measure the magnitude of an angle. In Geometry one-fourth of the whole circumference is taken as the unit called a quadrant or a right angle. That is to say that the arms of a right angle intercept one-fourth of the circumference. In Trigonometry a right angle is further divided to 90 parts called degrees. A degree is sub-divided to 60 minutes and a minute to 60 seconds. The symbols used for degree, minute and second are °, ′, ″, as 38° 12′ 53″, 215° 48′ 15″, etc. etc.



6. A right angle can be drawn with the help of a pair of set squares. The figures in the margin resemble set squares.



7. To draw a perpendicular on AB from the point C, hold one set square with one of its edges along the line AB as X, and place the other set square against it as Y and slide the latter until point C is reached. Now, by a lead pencil draw the line CD along the edge of the set square, this line CD is perpendicular on AB at C. In a similar manner a perpendicular can be drawn on a straight line from a point outside it.



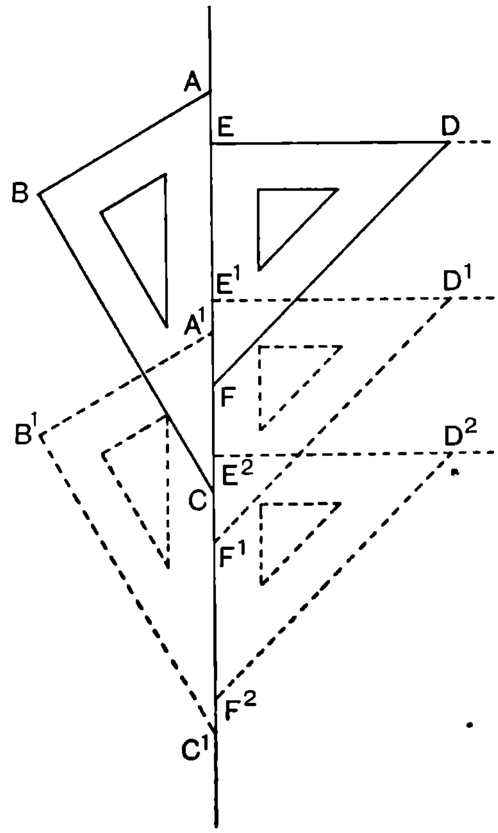
8. Parallels are drawn by set squares in the following manner :

Supposing a parallel to straight line ED is required to be drawn through the point E^1.

Hold the set square DEF with one edge along DE. Now hold the set square ABC against it in the position shown.

Now, holding ABC firm, slide EDF downward until the edge reaches E^1. Now hold the set square $D^1E^1F^1$ with an even pressure of the hand and draw the straight line E^1D^1 which is parallel to DE.

If a parallel is required to be drawn through E^2 then hold the set square in the position $D^1E^1F^1$ with a firm hand, and slide the other set square down

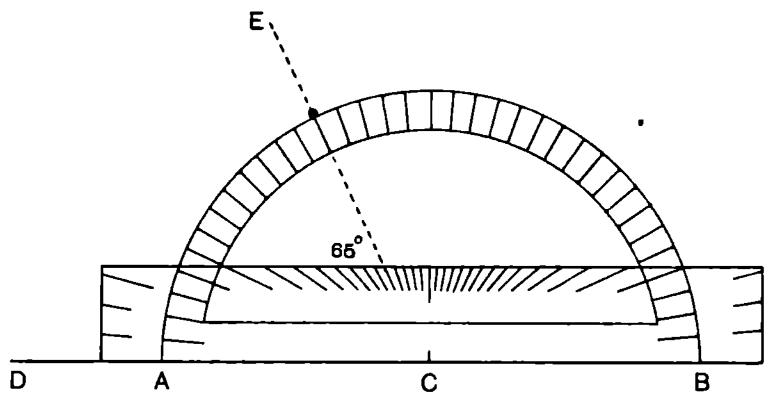


as $A^1B^1C^1$, then hold the $A^1B^1C^1$ firm and slide $D^1E^1F^1$ to the position $D^2E^2F^2$.

Straight lines DF, D^1F^1 and D^2F^2 are also parallel to one another.

In addition to drawing a right angle the angles of 60°, 30°, 45°, 75°, and 15° can be drawn by means of the set squares.

9. Protractor : For measuring angles of any magnitude a protractor is used. The figure below shows two forms of it, the semi-circular and the rectangular. To use the protractor the edge AB is held against one arm of the angle with the point C at the vertex, then a pencil dot is marked on the paper at the required graduation, after which the protractor is removed and the dot is joined with the vertex.



In the figure, angle DCE = 65°.

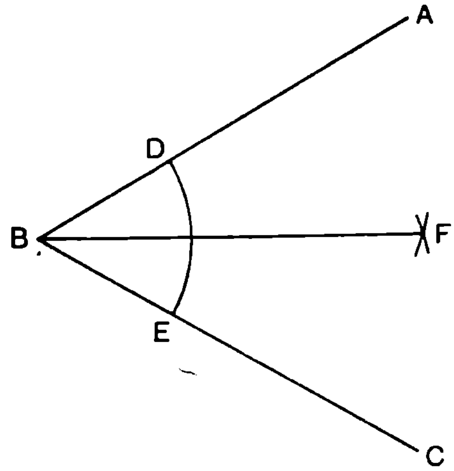
10. For measuring length of a line on paper, wooden or metal rules, called scales, with the marks of tenths or other fractions of inches are used. In some set squares the edges are marked with inches and fractions which can also be used.

11. Divider or compasses is used for taking measurements from the scales and for similar other purposes. This instrument is so simple that the very sight of it will suggest in how many different ways it can be used.

CHAPTER III

PRACTICAL GEOMETRY

12. Bisect the angle ABC.

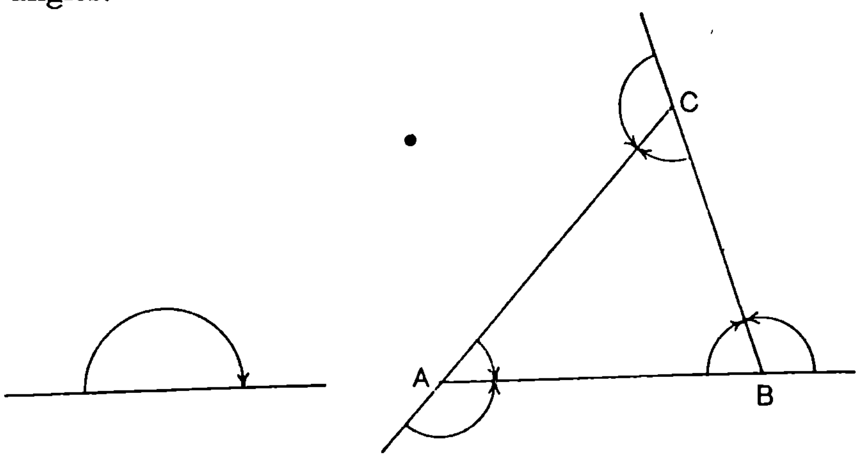


With the divider set off equal lengths BD and BE on the arms AB and BC respectively. With D as centre and any radius greater than half of DE draw an arc. With E as centre and with the same radius draw another arc cutting the previously drawn one at F. Join BF which bisects the angle ABC.

The angle FBC may also be bisected in a similar manner and thus one-fourth of the angle ABC obtained and so on.

13. Measure the sum of the angles of a triangle :

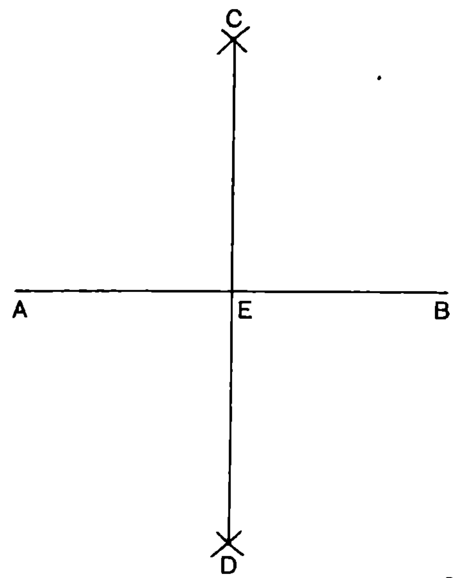
If a pencil is turned on a straight line end for end the pencil must have moved through half the circle, that is through two right angles.



Place a pencil on BC and turn it on the vertex C through the angle ACB ; slide the pencil along CA and turn the pencil through the angle CAB ; slide the pencil along AB and turn through the angle ABC. It will now be seen that the pencil has turned end for end on the line BC, that is to say that the pencil has traversed through a total angle of two right angles.

14. The sum of the three exterior angles of a triangle is equal to four right angles.

Starting from C, as in the previous case, turn the pencil through the exterior angles in the direction indicated by the arrows, when the pencil has come back along the side BC after traversing through the three exterior angles it has traversed through a complete revolution, that is to say, that the pencil has traversed through a total angle of four right angles.

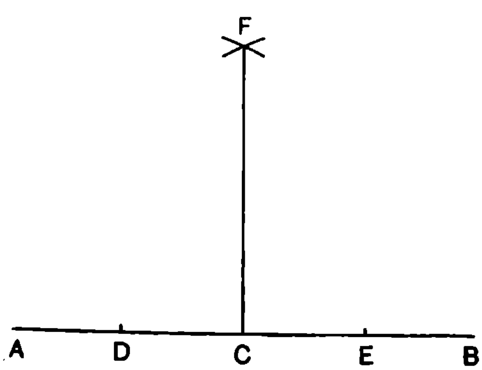


In the same way it can be seen that the sum of the exterior angles of any rectilineal figure is four right angles.

The interior or exterior angles may also be measured by means of a protractor and the result verified.

15. Bisect the given straight line AB.

With centre A, and any length of radius estimated to be more than half the length of AB, describe a circle. With centre B and the same length of radius describe another circle cutting the first at C and D. Join CD, cutting AB at E. AB will be bisected at E.



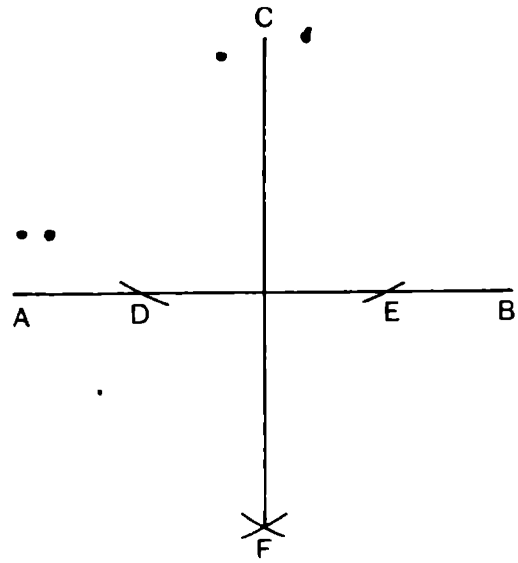
16. Draw a perpendicular on AB from a given point C in AB.

For the practical method of drawing a perpendicular with the help of a pair of set squares refer to Article 7, Chapter II.

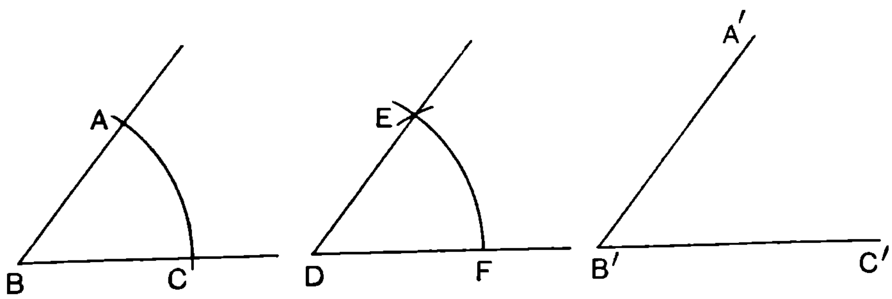
Take any length not more than CA in your compasses and cut off CD and CE equal to that. With centres D and E and any radius longer than DC draw two circles cutting one another at F. Join FC, which will be at right angles to AB.

17. Draw a perpendicular on AB from a point C not in AB or AB produced.

With C as centre and any radius describe a circle cutting AB at D and E. With centres D and E and any radius longer than half DE draw two circles cutting one another at F. Join CF which will be a perpendicular on AB.



18. Draw an angle equal to a given angle ABC.



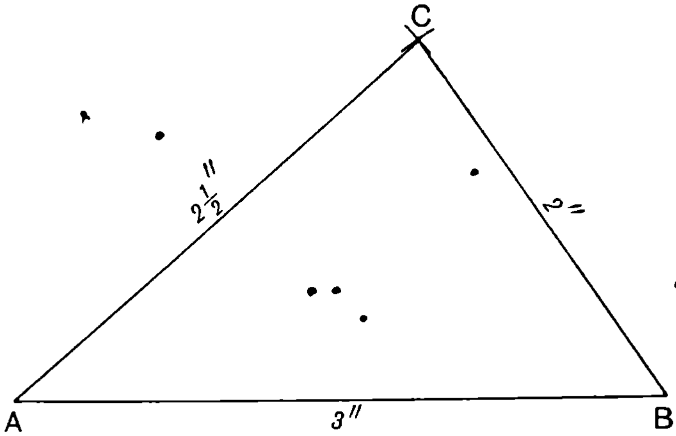
Practical method :

By means of a pair of set squares draw A^1B^1 parallel to AB and B^1C^1 parallel to BC. The angle $A^1B^1C^1$ is equal to the angle ABC.

Geometrical method :

Take any straight line DF. By means of a divider set off equal lengths BA, BC and DF. With F as centre and radius equal to AC draw a circle. With centre D and radius DF draw another circle cutting the former at E. Join DE. Angle EDF is equal to the angle ABC.

19. Construct a triangle having given the three sides as 2 in., $2\frac{1}{2}$ in. and 3 in.:



Take the length of 3 inches on your divider from a scale and mark AB 3 inches long. With A as centre and a radius $2\frac{1}{2}$ inches long draw a circle. With B as centre and a radius 2 inches long draw another circle cutting the previous one at C. Join AC and BC.

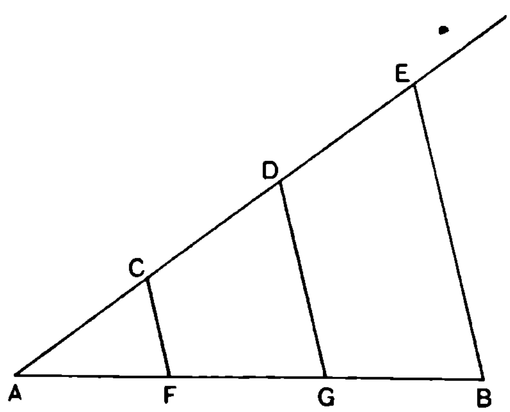
Triangle ABC is the required triangle, the sides measuring as under:

$$AB = 3 \text{ in.}$$
$$AC = 2\tfrac{1}{2} \text{ in.}$$
$$BC = 2 \text{ in.}$$

20. Divide the straight line AB into three equal parts. Draw AE making an acute angle with AB. Take any length AC on your divider and place it on the line AE as AC, CD, and DE. Join EB, and through C and D, draw parallels CF and DG meeting AB at F and G.

AB will be divided into three equal parts at F and G.

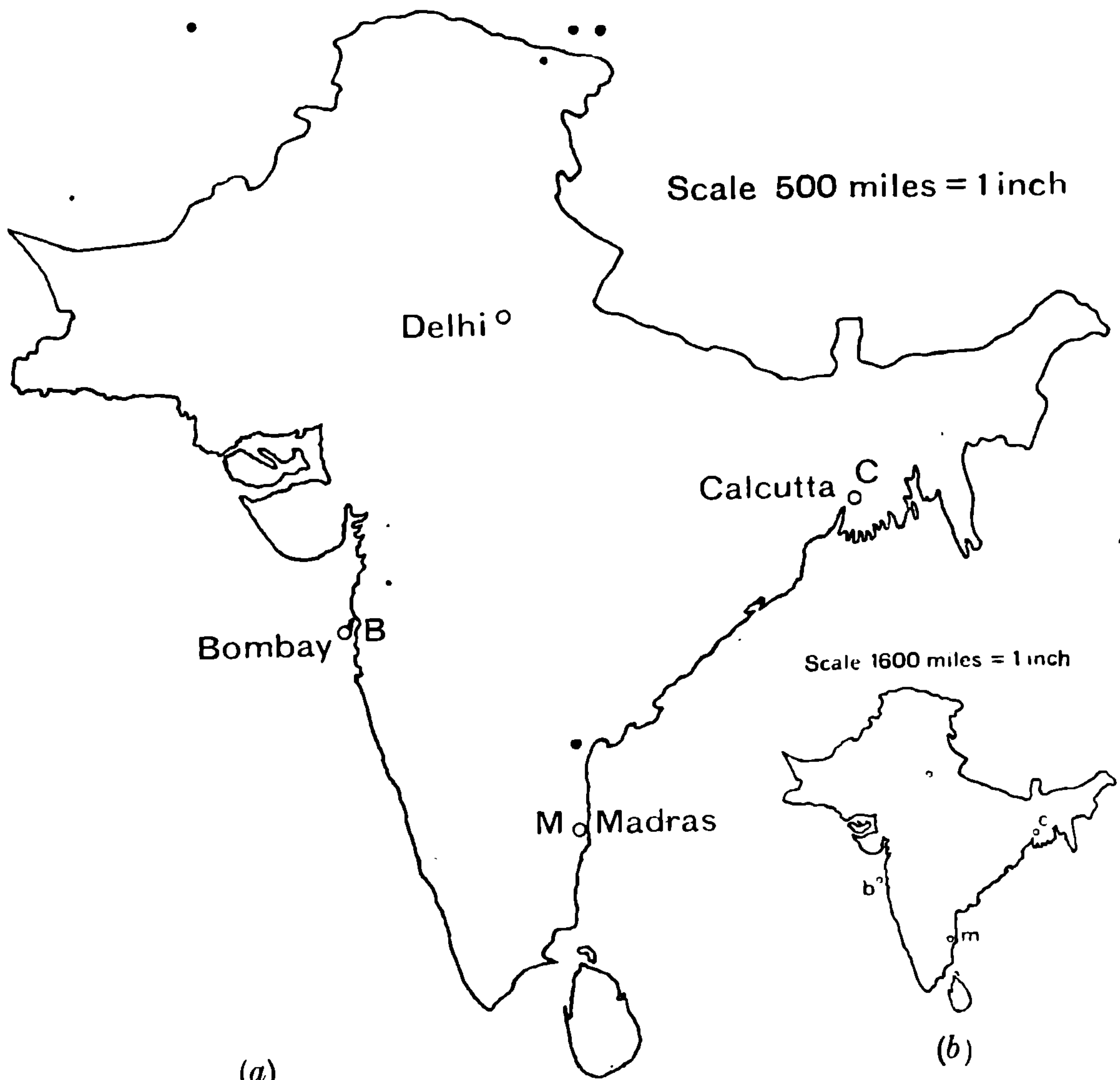
In the same way by placing 4, 5, 7, or any number of equal lengths on AE we can divide AB into 4, 5, 7, or any number of equal parts.



Chapter IV

SIMILAR FIGURES

21. When the outline or form of two figures resembles one another we speak of them as having the same shape even if they are of different sizes.



The above figures show two maps of India. Fig. (*a*) has been drawn to such a proportion that a length of one inch on it represents a length of 500 miles on the ground while a length of one inch in fig. (*b*) represents 1600 miles on the ground. This proportion is the scale of the map, or in other words fig. (*a*) is a map drawn to a scale of 500 miles to an inch and fig. (*b*), of 1600 miles to an inch.

The positions of three towns Calcutta, Madras and Bombay are marked on both maps. These positions are indicated by the letters C M B in fig (*a*), and by c m b, in fig. (*b*). The actual distance between Calcutta and Bombay, between Calcutta and Madras, or between Madras and Bombay, may be obtained from either map by measuring with a pair of compasses the distances between them on the scale of the map. Thus the distance between Calcutta and Bombay is 1023 miles, between Calcutta and Madras 836 miles, and between Madras and Bombay 638 miles.

$$\text{In fig. } (a) \ CB = \frac{1023}{500} \text{ inches}$$

$$\text{In fig. } (b) \ cb = \frac{1023}{1600} \text{ inches}$$

$$\text{or } \frac{CB}{cb} = \frac{500}{1600}$$

From the above it follows that the ratio of the distances (on paper) is the ratio of the scales of the maps.

If CB, BM, MC, and cb, bm and mc are joined, we can find, with the aid of a protractor, that the angle CBM is equal to the angle cbm, the angle BMC is equal to the angle bmc, and the angle MCB is equal to the angle mcb.

The two maps are similar figures and we arrive at the following properties of two similar figures:

(i) The ratio of the distance between any two points in the one figure to the distance between the corresponding points in the other figure is the same whatever points are taken.

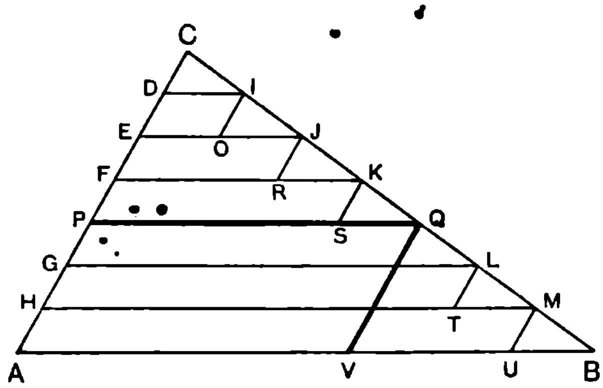
(ii) The angle between any two lines in the one figure is equal to the angle between the corresponding lines in the other figure.

22. The following propositions concerning similar figures are important.

(1) If a straight line PQ is drawn parallel to the side AB of the triangle ABC it cuts the other sides CA and CB in the same ratio.

Let us suppose that AP and CP are in the ratio of 3 to 4, that is to say that AP contains 3 of 7 parts of AC, and CP, 4.

Divide CP into 4 equal parts and AP into 3 equal parts. Draw through the points of divisions DI, EJ, FK, GL and HM parallel to AB.



Draw IO, JR, KS, etc., parallel to AC. Since DEOI, EFRJ, etc., are parallelograms and CD, DE, EF, etc., are equal to each other, CD is equal to IO, JR, KS, etc.

Now, the triangles CDI, IOJ, JRK, etc., are equiangular and have a corresponding side in each equal. Therefore, all these triangles are equal to one another in all respects. And CI, IJ, JK, KQ, etc., are equal to one another. Hence CB is also divided into 7 equal parts of which CQ contains 4 and QB 3.

$$\therefore\ CQ : QB :: 4 : 3$$

$$\therefore\ \frac{CQ}{QB} = \frac{4}{3}$$

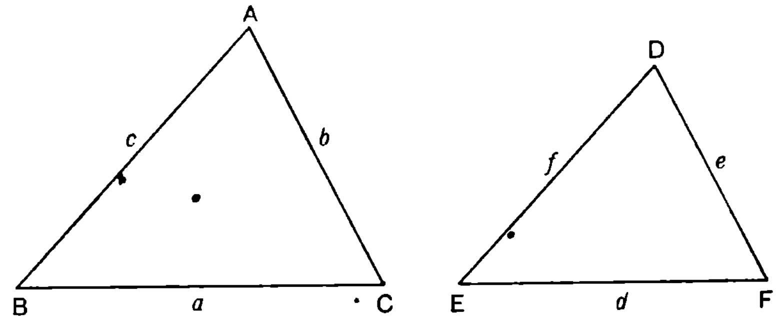
$$\text{But } \frac{CP}{PA} = \frac{4}{3} \qquad \therefore\ \frac{CP}{PA} = \frac{CQ}{QB}$$

If QV is drawn parallel to CA, as in the figure, it can be shown in the same way that

$$\frac{AV}{BV} = \frac{CQ}{BQ} = \frac{4}{3}$$

each of the triangles CPQ and QVB is also similar to the triangle CAB.

The ratio of each side of CPQ to the corresponding side of CAB is 4 to 7, while the ratio of each side of QVB to the corresponding side of CAB is 3 to 7.



It is important to notice that the corresponding sides of similar triangles are opposite equal angles.

We arrive at the following conclusions from the above. The triangles ABC and DEF are similar if

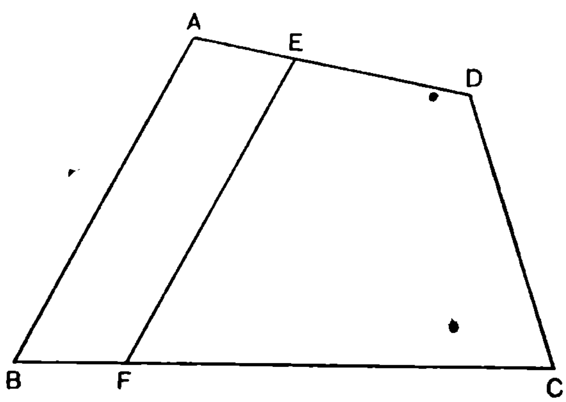
(i) $\frac{a}{d}=\frac{c}{f}=\frac{b}{e}$

(ii) the triangles ABC and DEF are similar if $\angle A = \angle D$, $\angle B = \angle E$; $\angle C = \angle F$.

(If two angles in one triangle are equal to two angles in the other, the third angles must of course be equal.)

(iii) The triangles ABC and DEF are similar if the ratio of a pair of sides in one is equal to the ratio of a pair of sides in the other and the angles included in the pairs of sides are equal, i.e., if

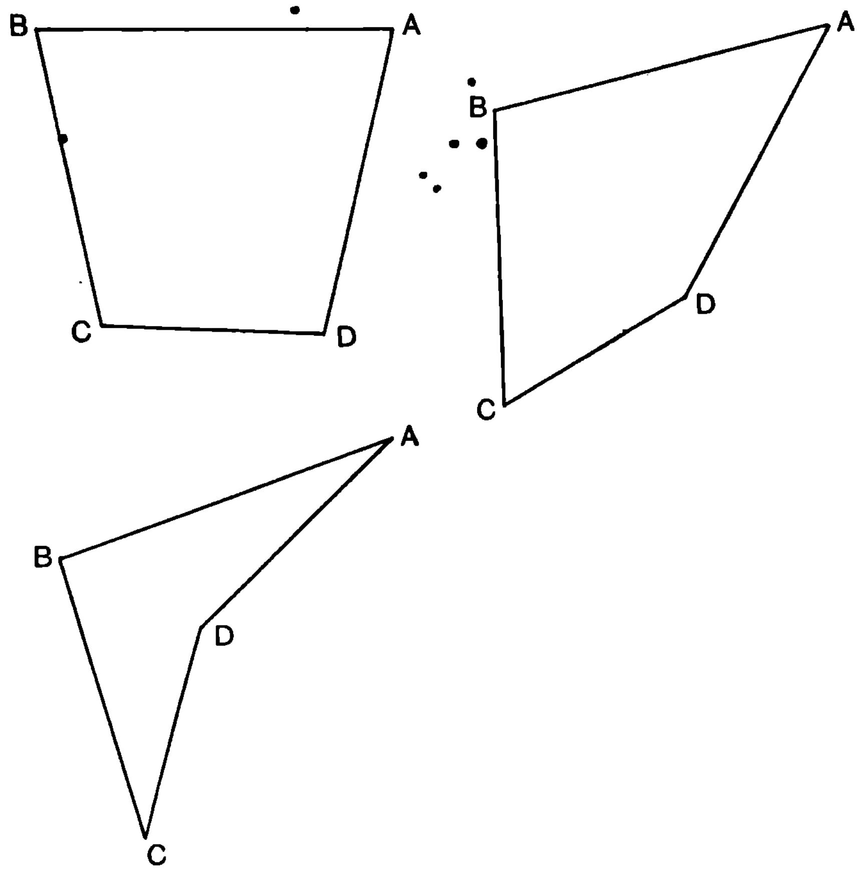
$$\frac{a}{b}=\frac{d}{e} \text{ and } \angle C = \angle F$$



23. Two triangles are similar if their angles are equal; but polygons formed of four or more sides may not be similar if their angles are equal.

In the fig., EF is drawn parallel to AB. The figures ABCD and EFCD are equiangular. But the ratio CD : CB cannot be equal to the ratio CD : CF. Therefore, though the quadrilaterals ABCD and EFCD are equiangular they are not similar.

Two triangles are also similar if their sides are in the same ratio; but polygons formed of four or more sides may not necessarily be similar even if their sides are in the same ratio. This will be clear from the fact that a frame made of three bars jointed at their ends is a stiff frame. But a frame formed by four jointed bars as ABCD may be distorted into numerous shapes:

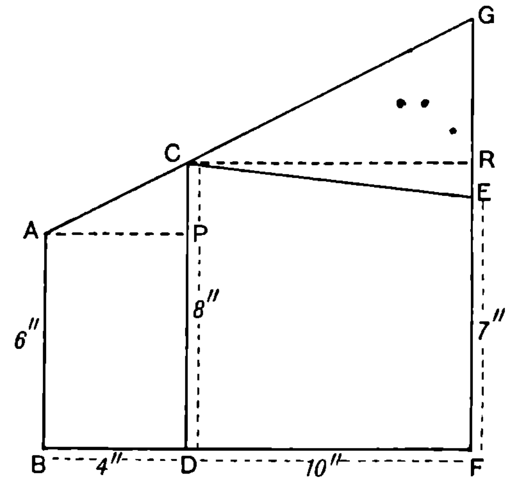


[It is interesting to note that the properties of similar triangles were applied by the Greek mathematician Thales (640–550 B.C.), who is regarded as the founder of the earliest Greek school of mathematics and philosophy. Plutarch, the biographer (A.D. 46–120) recorded that Thales measured the height of one of the Pyramids of Egypt by erecting a stick of known length and measuring the length of the shadow of the stick cast by the sun, and at the same time the length of the shadow of the Pyramid, thus:

$$\frac{\text{Height of Pyramid}}{\text{Length of shadow of Pyramid}} = \frac{\text{Length of stick}}{\text{Length of shadow of stick}}$$

"The King Amasis, who was present, is said to have been amazed at this application of abstract science, and the Egyptians seem to have been previously unacquainted with the theorem."]

Illustrated Examples



Ex. 1. AB, CD and EF are three perpendiculars on BF. AC is produced to meet FE produced in G. Find the length of EG, given AB = 6 in., CD=8 in., and FE=7 in., BD=4 in., and DF=10 in.

Draw AP and CR parallel to BF.

Now, the triangles CAP and GCR are similar.

$$\therefore \frac{CP}{AP} = \frac{GR}{CR}$$

BD = AP = 4 in.; CP = CD−DP (AB) = 8−6 = 2 in.
CR = DF = 10 in.

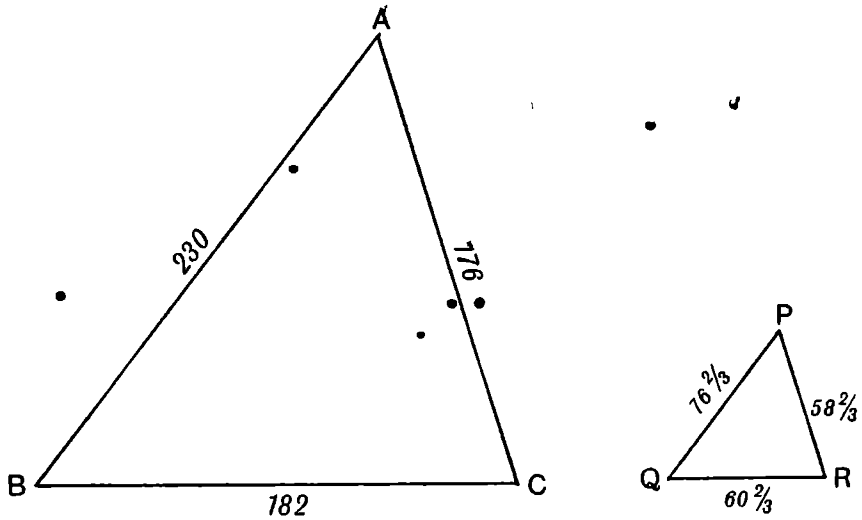
$$\therefore \frac{CP}{AP} = \frac{2}{4} = \frac{GR}{10 \text{ in.}}$$

$$\therefore GR = \frac{2}{4} \times 10 \text{ in.} = 5 \text{ in.}$$

RE = RF−EF; But RF = CD = 8 in.
∴ RE = 8 in.−7 in. = 1 in.
∴ EG = 5+1 = 6 in.

Ex. 2. ABC and PQR are two similar triangles. ∠A=∠P and ∠B = ∠Q.

AB is 230 ft., BC 182 ft., AC 176 ft. The perimeter of the triangle PQR 196 ft. Find the sides of PQR.



Perimeter of ABC = 230+182+176 = 588 ft.

$$\frac{PQ}{AB} = \frac{196}{588}$$

$$\text{or } PQ = \frac{\not{1}\not{9}\not{6}}{\underset{3}{\not{5}\not{8}\not{8}}} \times 230 = \frac{230}{3} = 76\tfrac{2}{3} \text{ ft.}$$

$$\frac{QR}{PQ} = \frac{BC}{AB}$$

$$\therefore\ QR = \frac{182}{\not{2}\not{3}\not{0}} \times \frac{\not{2}\not{3}\not{0}}{3} = \frac{182}{3} = 60\tfrac{2}{3} \text{ ft.}$$

$$\text{and } PR = 196-(76\tfrac{2}{3}+60\tfrac{2}{3}) = 58\tfrac{2}{3} \text{ ft.}$$

Exercise 1

In the following exercises ABC and PQR are two similar triangles ; angles are denoted by the capital letters and the corresponding sides by small letters. $\angle A = \angle P$; $\angle B = \angle Q$; $\angle C = \angle R$.

1. Find the remaining elements from the following :

(i) $a = 15$; $b = 19$; $c = 23$; $q = 11$
(ii) $a = 4{\cdot}2$; $c = 7{\cdot}6$; $p = 5{\cdot}25$; $q = 3{\cdot}6$
(iii) $a+b = 60$; $c = 38$; $r+q = 13{\cdot}5$; $p = 11$

2. ABCD is a rectangle having AB = 11 in., AD = 16 in E is a point in AD such that ED = 3 in. O is the point of intersection of the diagonals. EO produced meets AB produced in F. Find the length of BF.

3. ABC is a triangle in which AB = 14 in., AC = 12 in., BC = 16 in. P is a point in BC such that BP = 3 in. PQ is drawn parallel to AB meeting AC in Q. QR is drawn parallel to AP meeting BC in R. Find the length of PR.

4. A tower casts a shadow of 189 ft. when a vertical pole 10 ft. long casts a shadow 15 ft. 9 in. long. What is the height of the tower?

5. (i) A man 5 ft. 9 in. in height, standing 4 ft. from a lamp-post, notices that his shadow is at right angles to a wall and just reaches it. How far is the wall from the lamp-post, if the light is 10 ft. from the ground?

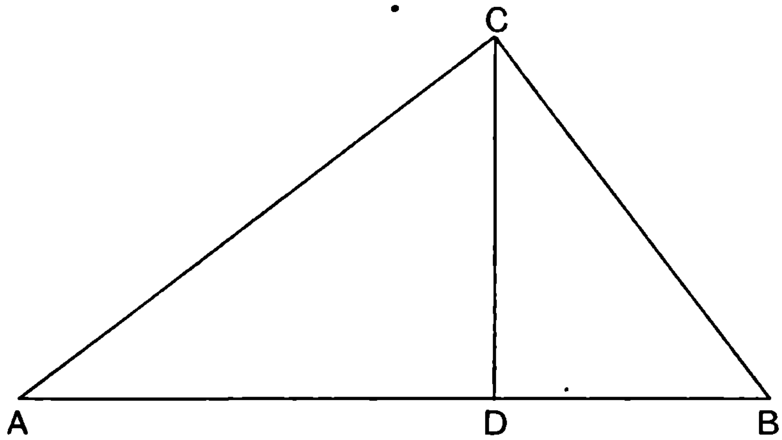
(ii) Find how high up on the vertical wall his shadow would reach if he were to stand 6 ft. from the lamp-post.

CHAPTER V

RIGHT ANGLED TRIANGLE

24. Pythagoras' Theorem*

The square on the hypotenuse of a right angled triangle is equal to the sum of the squares on the sides containing the right angle.



In the triangle ABC, the angle ACB is a right angle. Draw the perpendicular CD on AB. Then the triangles ABC and ADC are similar, because the angle A is common and the angle ACB = ADC being right angles.

$$\therefore \frac{AB}{AC}=\frac{AC}{AD} \text{ or } AB \times AD = AC^2 \qquad \text{(i)}$$

Again the triangles CDB and ACB are similar

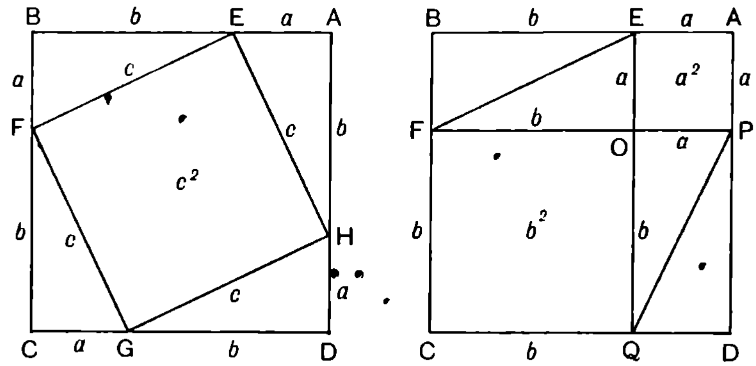
$$\therefore \frac{AB}{CB}=\frac{CB}{DB} \text{ or } AB \times DB = CB^2 \qquad \text{(ii)}$$

Adding equations (i) and (ii) together we get

$$\begin{aligned} AC^2 + CB^2 &= (AB \times AD)+(AB \times DB) \\ &= AB\,(AD+DB) \\ &= AB \times AB \\ &= AB^2. \end{aligned}$$

* Pythagoras (569–500 B.C.) was the founder of a famous school of mathematics and philosophy. But the theory was known to the Chinese as far back as 2637 B.C.

As Pythagoras' theorem is very important in mensuration, a geometrical independent proof is added below :



(Fig. 1.) (Fig. 2.)

Take any square ABCD, and in its sides (in Fig. 1) cut off AE = BF = CG = DH = say, a. Join EF, FG, GH and HE. EFGH will be a square. Let EF = c, then EFGH = c^2.

Also BE = FC = GD = AH = say, b.

The four triangles EBF, FCG, GDH and AEH are equal in all respects to each other.

In the square ABCD in Fig. 2

cut AE = AP = a

Draw EQ and PF parallel to AD and AB respectively.

The triangles FBE, FOE, OPQ and PQD are equal to each other in all respects as also each of them is equal in all respects to each of the triangles EBF, etc., in Fig. 1.

In Fig. 2

AEOP = a^2 ; OFCQ = b^2

Sq. ABCD in Fig. 1

= c^2+triangle EBF+triangle FCG+triangle GDH +triangle HEA

In Fig. 2

Sq. ABCD = a^2+b^2+triangle BFE+triangle FEO+triangle OQP+triangle PQD

$$\therefore\ c^2 = a^2+b^2.$$

Important.

A triangle whose sides are 3, 4, 5 is right angled, for,

$$3^2+4^2 = 5^2$$

A triangle whose sides are 5, 12, 13 or 7, 24, 25 is a right angled triangle.

Illustrated Examples

Ex. 1. In a right angled triangle the sides containing the right angle measure 11 yd. and 18 yd. 2 ft. respectively. Find the length of the hypotenuse.

11 yd. = 33 ft.; 18 yd. 2ft. = 56 ft.

$$\text{Hypotenuse} = \sqrt{33^2+56^2} = \sqrt{1089+3136} = \sqrt{4225}$$
$$= 65 \text{ ft., or } 21 \text{ yd. } 2 \text{ ft.}$$

Ex. 2. In a right angled triangle the hypotenuse is 15 ft. 5 in. and one side is 8 ft. 8 in. Find the other side.

$$\left.\begin{array}{r} 15 \text{ ft. } 5 \text{ in.} = 185 \text{ in.} \\ 8 \text{ ft. } 5 \text{ in.} = 104 \text{ in.} \end{array}\right\}$$

$$\begin{aligned} \text{Required side} &= \sqrt{185^2-104^2} \\ &= \sqrt{(185+104)\ (185-104)} \\ &= \sqrt{289\times81} \\ &= \sqrt{17\times17\times9\times9} \\ &= 17\times9 \\ &= 153 \text{ in.} \\ &= 12 \text{ ft. } 9 \text{ in.} \end{aligned}$$

Ex. 3. The side of a square measures 3 ft. 4 in., find the length of its diagonal.

The diagonal of a square is the hypotenuse of an isosceles right angled triangle whose sides are the sides of the square.

(3 ft. 4 in. = 40 in.)

$$\begin{aligned} \therefore \text{ Diagonal} &= \sqrt{\text{side}^2+\text{side}^2} \\ &= \sqrt{2\ \text{side}^2} \\ &= \text{side } \sqrt{2} = 40 \text{ in.} \times 1{\cdot}41421 \\ &= 56{\cdot}5684 \text{ in.} \end{aligned}$$

Important.

The students should carefully remember the following:

$$\sqrt{2} = 1{\cdot}41421\ ; \quad \sqrt{3} = 1{\cdot}73205\ ; \quad \sqrt{6} = 2{\cdot}44949.$$

Ex. 4. The hypotenuse of an isosceles right angled triangle is 5 chains 20 links. Find the length of a side.

$$\text{Side} = \frac{5 \cdot 20}{\sqrt{2}}$$
$$= \frac{5 \cdot 20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{5 \cdot 20 \sqrt{2}}{2}$$
$$= 2 \cdot 60 \sqrt{2}$$
$$= 3 \cdot 6769 \text{ chains.}$$

Ex. 5. The difference between the hypotenuse and the base of a right angled triangle is 2 in., and the perpendicular is 16 in. Find the hypotenuse and the base.

Let hypotenuse $= x$

Then base $= (x-2)$ in.

But perpendicular $^2 =$ hypotenuse$^2 -$ base^2

$\therefore \quad 16^2 = x^2-(x-2)^2$

or $\quad 256 = x^2-(x^2+4-4x)$

or $\quad 256 = x^2-x^2-4+4x$

or $\quad 256+4 = 4x$

$\therefore \quad x = \frac{260}{4} = 65$ in.

$\therefore$ hypotenuse $= 65$ in.

and the base $= 65-2 = 63$ in.

Ex. 6. The sum of the hypotenuse and perpendicular of a right angled triangle is 64 in., and the base is 48 in. Find the hypotenuse and perpendicular.

Let hypotenuse $= x$ in.

then $48^2 = x^2-(64-x)^2$

$= x^2-4096-x^2+128x$

or $2304+4096 = 128x$

or $\frac{6400}{128} = x$

$\therefore x = 50$ in.

i.e., hypotenuse $= 50$ in. and perpendicular $= 64-50 = 14$ in.

Ex. 7. The hypotenuse of a right angled triangle measures 97 in., and the sum of the two other sides is 137 in. Find the sides.

Let the sides be a and b

$$\text{then } a+b = 137 \text{ in.}$$

$$\text{Now } (a+b)^2+(a-b)^2 = 2(a^2+b^2)$$
$$\text{but } a^2+b^2 = 97^2$$

$$\text{or } 137^2+(a-b)^2 = 2(97^2)$$
$$\therefore (a-b)^2 = 2(97^2)-137^2$$
$$= 18818-18769$$
$$= 49$$

$$\therefore a-b = \sqrt{49} = 7$$

$$\text{But } a+b = 137$$
$$a-b = 7$$

$$\therefore a = \frac{144}{2} = 72 \text{ in.}$$
$$\text{and } b = 137-72$$
$$= 65 \text{ in.}$$

Ex. 8. In a right angled triangle the difference of the two sides is 21 ft., and the hypotenuse is 39 ft. Find the sides.

Let the sides be a and b,

then a—b = 21 ft.

$$\text{Now } (a+b)^2+(a-b)^2 = 2(a^2+b^2)$$
$$\text{but } a^2+b^2 = 39^2$$

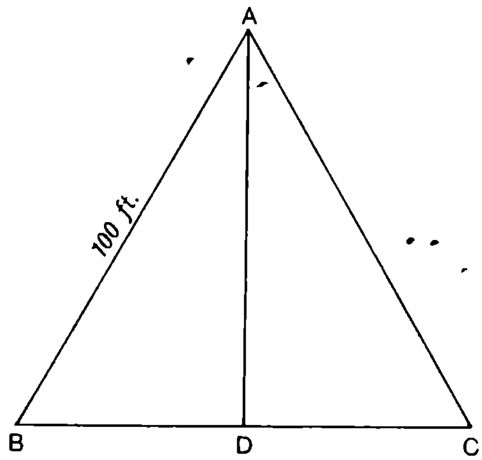
$$\therefore (a+b)^2+21^2 = 2(39^2)$$
$$\text{or } (a+b)^2 = 2(39^2)-21^2$$
$$\text{or } (a+b)^2 = 3042-441$$

$$\text{or } a+b = \sqrt{2601}$$

$$\therefore a+b = 51$$
$$\text{and } a-b = 21$$

$$\therefore a = 36 \text{ ft.}$$
$$\text{and } b = 36-21$$
$$= 15 \text{ ft.}$$

Ex. 9. The side of an equilateral triangle is 100 ft. Find the length of the perpendicular from any angular point to the opposite side.



Let the side be a

then $BD = \frac{a}{2}$

$$AD = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$$

$$= \sqrt{a^2 - \frac{a^2}{4}}$$

$$= \sqrt{\frac{3a^2}{4}}$$

$$= \frac{a\sqrt{3}}{2}$$

That is to say that the height of an equilateral triangle

$$= \text{side} \times \frac{\sqrt{3}}{2}$$

in this case

$$\therefore \text{ height} = 100 \times \frac{\sqrt{3}}{2} = 86{\cdot}6 \text{ ft.}$$

From the above we can further contend that in the right angled triangle ABD

$\angle ABD = 60°$
$\angle BAD = 30°$

and the sides bear the ratios of

$$\left\{\begin{matrix}\text{AB}\\\text{the}\\\text{hyp.}\end{matrix}\right\} : \left\{\begin{matrix}\text{AD}\\\text{opp. to}\\60°\end{matrix}\right\} : \left\{\begin{matrix}\text{BD}\\\text{opp. to}\\30°\end{matrix}\right\} :: 1 : \frac{\sqrt{3}}{2} : \frac{1}{2}$$

Ex. 10. A, B, and C are three places situated such that AB is 12 miles, AC is 14 miles and the angle BAC is 120°; find the direct distance between B and C.

Draw perpendicular CD on BA produced.

In the right angled triangle CAD

$\angle CAD = 60°$,

$\angle ACD = 30°$

$\therefore\ AD = \frac{AC}{2}$

$= \frac{14}{2} = 7$ miles

and $CD = 14\frac{\sqrt{3}}{2}$

$= 7\sqrt{3}$ miles

$BD = BA + AD$

$= 12 + 7$

$= 19$

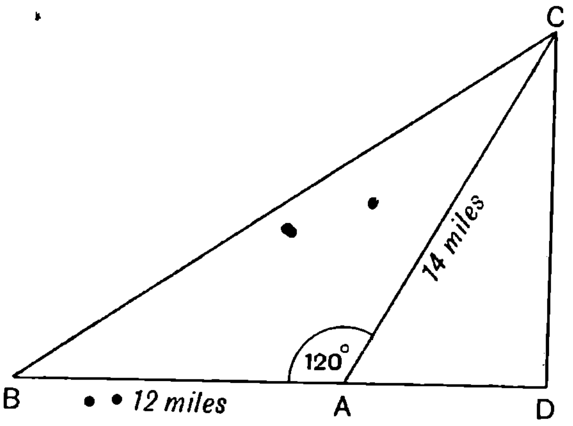
Now $BC = \sqrt{BD^2 + CD^2}$

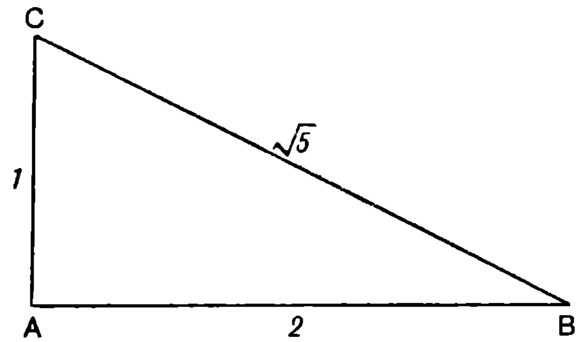
$= \sqrt{19^2 + (7\sqrt{3})^2}$

$= \sqrt{361 + 147}$

$= \sqrt{508}$

$= 22{\cdot}538$ miles.





Ex. 11. Draw a straight line $\sqrt{5}$ in. long.

Take AB 2 in. long; at A on AB draw the perpendicular AC 1 in. long.

Join BC. BC is $\sqrt{5}$. in.

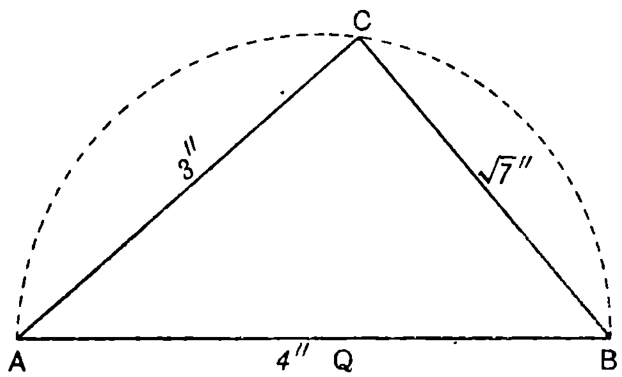
Ex. 12. Draw a straight line $\sqrt{7}$ in. long.

Take AB 4 in. long.

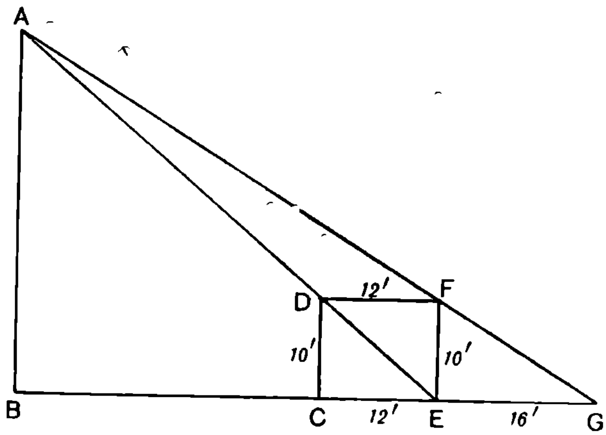
With AB as diameter draw the semicircle ACB.

With A as centre and a radius 3 in. in length draw a circle cutting the previous one at C.

Join CB. CB is $\sqrt{7}$ in.



Ex. 13. In the diagram AB is a tower. A rope AE touches the top of a vertical pole CD at 12 ft. from E (that is EC = 12 ft.). Another rope AG touches the top of the pole CD removed and placed vertically at E. EG = 16 ft. and CD = 10 ft. Find BC and AB.



In the diagram the triangles ABE and DCE are similar.

Hence AB : BC+CE :: CD : CE (i)

Again the triangles ABG and FEG are similar.

Hence AB : BC+CE+EG :: CD (FE) : EG (ii)

or BC+CE+EG : BC+CE :: EG : CE

or CE (BC+CE+EG) = EG (BC+CE)

or $(CE \times BC) + CE^2 + (CE \times EG) = (BC \times EG) + (CE \times EG)$

or $CE^2 = BC\,(EG - CE)$

or $$\frac{CE^2}{EG - CE} = BC$$

Here $$BC = \frac{12^2}{16 - 12} = 36 \text{ ft.}$$

To find AB

CE : CD :: BE : AB

or $$AB = \frac{CD \times BE}{CE}$$

Here $$AB = \frac{10 \times 48}{12}$$

$$= 40 \text{ ft.}$$

AB can also be found out in the following manner:

AB : BC+CE :: CD : CE

or $AB \times CE = (BC \times CD) + (CE \times CD)$ (i)

Again AB : BC+CE+EG :: CD : EG

or $AB \times EG = (BC \times CD) + (CE \times CD) + (EG \times CD)$ (ii)

Subtracting (i) from (ii) we have

$(AB \times EG) - (AB \times CE) = EG \times CD$

or $AB\ (EG - CE) = EG \times CD$

or $AB = \dfrac{EG \times CD}{EG - CE}$

Here $AB = \dfrac{16 \times 10}{16 - 12} = 40$ ft.

Exercise 2

In the following exercises a, b, c denote the sides opposite the angles A, B, C, respectively in the right angled triangle ABC, angle C being the right angle.

1. Given a = 40 ft., b = 9 ft., find c.
2. Given c = 10 in., a = 6 in., find b.
3. Given a = 7 in., A = 45°, find c.
4. Given A = 30°, c = 10 ft., find a and b.
5. Given c = 3·24 chains, a = 2·72 chains, find b.
6. Given c = 4 m. 5 fur., b = 4 m. 3 fur., find a.
7. Given c = 53 rasi, a = 45 rasi, find b.
8. Given a = 85 ft., b = 132 ft., find c.
9. Given b = 88 m., c = 137 m., find a.
10. Given c = 123 ft., a = 9 yd., find b.
11. The diagonal of a square is 10 ft. Find the length of a side correct to three decimal places. Also give the result to the nearest inch.
12. A rectangular field measures 7 chains in length and 3 chains in breadth. Find the length of the straight line joining the two opposite corners.
13. The side of an equilateral triangle is 7 ft. long. Find the altitude.

14. The altitude of an equilateral triangle is 5 ft. Find the side.

15. Find the altitude of an isosceles triangle whose equal sides are 13 in., and the base 24 in. long.

16. A place P lies 40 miles west of a place Q, and 42 miles south of another place R. Find the distance of Q from R.

17. Construct geometrically the lengths of $\sqrt{5}$; $\sqrt{11}$; $\sqrt{13}$ in.

18. The span of a roof is 21 ft., and the rise 7 ft. Find the slope length of each side.

19. Find the cost of putting a fence round an enclosure in the shape of a right angled isosceles triangle whose hypotenuse measures 200 ft. at Re. 1 per foot.

20. The hypotenuse of a right angled triangle is 5 chains, and one side is half of the other. Find the sides.

(In the following exercises the same notations as for exercises 1 to 10 are followed.)

21. $a+c = 32$ ft.; $b = 8$ ft. Find a and c.

22. $a+c = 49$ ft.; $b = 21$ ft. Find a and c.

23. $c+b = 10$ ft. 1 in.; $a = 6$ ft. 5 in. Find c and b.

24. $c-a = 9$ in.; $b = 2$ ft. 9 in. Find c and a.

25. $c-b = 8$ yd. 1 ft.; $a = 21$ yd. 2 ft. Find c and b.

26. $c+a = 3$ chains 61 links.; $b = 2$ chains 9 links. Find a and c.

27. $c-a = 4$ chains 90 links.; $b = 9$ chains 10 links. Find a and c.

28. $a+b = 47$ ft.; $c = 37$ ft. Find a and b.

29. $a-b = 2$ ft. 10 in.; $c = 4$ ft. 2 in. Find a and b.

30. $a+b = 13$ yd. 2 ft.; $c = 9$ yd. 2 ft. Find a and b.

31. $a+b = 6$ chains 20 links; $c = 5$ chains. Find a and b.

32. $a-b = 2$ chains 30 links.; $c = 3$ chains 70 links. Find a and b.

33. $a+b = 5$ rasi 19 katha; $c = 4$ rasi 11 katha. Find a and b.

34. The hypotenuse of a right angled triangle is 210 yd. One side is longer by 42 yd. than the other. Find the sides.

35. A man starts from a town A and walks due east to a town B and thence walking due north reaches the town C in 10 hours and

12 minutes, walking throughout at a uniform speed of 5 miles per hour. He returns to A direct from C, walking at a similar speed, in 2 hours 24 minutes less than what he required to go to C. Find the distance of the towns A and C from B.

36. One side of a right angled triangle is 588 ft.; the sum of the hypotenuse and other side is 882 ft. Find the hypotenuse and the other side.

37. One side of a right angled triangle is 3925 ft.; the difference between the hypotenuse and the other side is 625 ft. Find the hypotenuse and the other side.

38. A ladder 27 ft. 1 in. long is placed leaning on a wall with the foot of it at a distance of 7 ft. 7 in. from the wall. If the foot of the ladder is drawn out 2 ft. 10 in. further, how far should the top of the ladder come down?

39. A ladder 25 ft. long is placed against a wall with its foot 7 ft. from the wall. How far should the foot be drawn out so that the top of the ladder may come down by half the distance that the foot is drawn out?

40. A tower which stands in a horizontal plane, subtends a certain angle at a point 160 ft. from the foot of the tower. On advancing 100 ft. towards it the tower is found to subtend an angle twice as great as before. What is the height of the tower?

41. Two roads diverge from a point at an angle of 120° to each other. Two persons, one on each road, start from the point at the rate of 4 and 5 miles per hour respectively. What will be the direct distance between the persons after they shall have travelled for 6 hours on their respective roads?

42. Given the perimeter of an isosceles right angled triangle $= \sqrt{2}+1$, find the hypotenuse.

43. ABC is a right angled triangle, right angled at B; D is a point in AB; $BD = BC = 33$ ft. $= \frac{1}{2}(AD+AC)$. Find AB.

44. A man on one side of a brook finds that he can just rest a ladder 20 ft. long against the branch of a tree standing vertically over the other bank; the branch is 12 ft. above the ground. How wide is the brook?

45. The breadth of the bottom of a ditch is to be 16 ft., and the depth 9 ft., and the inclinations of the sides to the top 30° and 45°. What must be the breadth of excavation at the top?

46. The span of a roof is 24 ft. and the rise 4 ft. Find the slope length of each side.

47. From a point within a rectangle, lines measuring 16 in. and 20 in. are drawn to opposite angles; a third line measuring 12 in. is also drawn to one of the other angles. Find the length of the line drawn from the point to the remaining angle.

48. In the middle of a pond 10 ft. by 10 ft. grew a reed which raised its head 1 ft. above the surface of the water. A person standing on the brink at a middle point of one of its sides could just draw the top of the reed to the edge of the bank. How deep was the water?

49. A river is 120 feet in breadth and a boat is being towed up the midstream by two men on opposite banks with strings 100 ft. and 156 ft. long. How far are the men apart?

50. The cable joining two consecutive telegraph posts is broken at 60 ft. from one post and the broken ends touch the ground at two points which are 16 ft. apart and are in the straight line joining the base of the posts. Supposing the posts are situated on a level piece of ground, are absolutely vertical, and of equal heights, and 224 ft. apart, find the height of the posts.

51. A man walks 11 miles north then 10 miles west and then 13 miles north again. What distance will he walk if he returns to the starting place direct?

52. Two towers 100 ft. and 50 ft. respectively in height are situated some distance apart. The top of each is joined by a rope with the base of the other. Find the height of the point at which the two ropes have crossed.

53. In a right angled triangle the distance of the right angle from the middle point of the hypotenuse is 20 ft. One side is 24 ft. Find the other side.

54. The two sides of a triangle are 25 ft. and 30 ft. respectively; the difference of the two sections of the base by the perpendicular drawn from the vertex is 11 ft. Find the base.

55. Two towers of equal height are situated on level ground some distance apart. An observer observes the altitudes of the top of each from the middle point of the line joining the bases to be 30°. After receding 40 ft. along the same line he observes the altitude of the nearer tower to be 60°. Find the height of the towers and the distance between them.

56. A ship sailing south-eastward visits a beacon in the north-east direction; after proceeding 4 miles on her way the beacon is seen to bear 15° east of the north. How far was the ship from the beacon on each occasion?

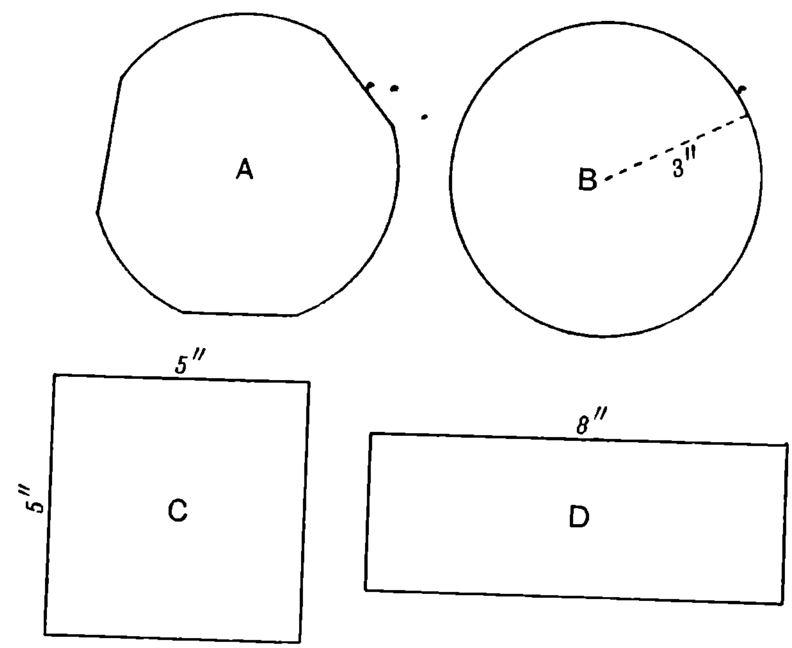
57. An observer observes the altitude of the top of a palm tree and a tower which are situated on a straight line from him to be 30° and 45° respectively; after advancing 100 ft. he observes the tops of the both in the same line and at an altitude of 60°. Find the distance of the tower from the tree and the height of each.

58. P and Q are inaccessible places. A man observes from a point A due west of Q that the line PQ subtends an angle of 30° to his eye. After removing to B which is due south of P and 120 yd. from A he observes that PQ still subtends 30° to his eye. Find the distance between P and Q.

59. A forester with a view to determine the height of a tree holds vertically a 15 ft. pole at some distance from the tree and after receding 12 ft. from the pole in a line, observes the top of the pole and the top of the tree in a straight line. He then removes the pole at this latter point and after placing it vertically there recedes 14 ft. further and observes the top of the pole and the top of the tree in a straight line. Taking the height of the forester's eye to be 5 ft., find the height of the tree and its distance from the first position of the pole.

CHAPTER VI

AREA



The above represents four pieces of cakes of equal thickness; which would you prefer to have? or, these represent four fields of equally good soil and equal rent; which would you choose?

The problem, how to measure a surface, definitely emerges from the above. It also leads up to the idea that for measuring a given surface a "measuring surface" is necessary; and a square is obviously a good shape for it.

Let us make a number of small paper squares of equal size and cover each of the figures, A, B, C, D with them. Now by counting the squares we can arrive at a satisfactory answer to the above question. Again we observe that C and D are most easily dealt with. D is a rectangle; suppose there are three rows of squares and in each row there are eight squares. Then the rectangle contains ($8 \times 3 =$) 24 squares. That is to say that the length gives the number of squares in a row; the breadth, the number of rows. The number of squares contained in the rectangle is obtained by multiplying the number of squares in each row by the number of rows. One square inch is the space covered by a square whose sides are one inch each. If the paper squares be one square inch and the rectangle D is 8 in. in length and 3 in. in breadth, then there will be three rows of 8 sq. in. or the area will be 24 sq. in. This result we arrive at by simply multiplying the number of inches in length by the number of inches in breadth. In brief we can put that the area of rectangle = length $\times$ breadth.

In a square the length and the breadth are equal, therefore, the area of a square is

length $\times$ length

or

side^2.

As has been said above one square inch is the space covered by a square whose side is one inch. One square foot is the square whose side is one foot and so on.

Now, 12 inches make one foot. So one square foot contains 12 rows, each row having 12 one inch squares. Therefore one square foot is equal to ($12 \times 12 =$) 144 square inches. Similarly one square yard is equal to ($3 \times 3 =$) 9 square feet, and so on.

26. The tables below give the square measures of different units :

English

12^2 or 144 sq. inches = 1 sq. foot

3^2 or 9 sq. feet = 1 sq. yd.

$5\frac{1}{2}^2$ or $30\frac{1}{4}$ sq. yards = 1 sq. pole

40 sq. poles = 1 rood

4 roods = 1 acre*

640 acres = 1 sq. mile

$\therefore$ One acre = 4840 sq. yd. = 10 sq. chains.

* Fractions of an acre are now usually counted in decimals and hundredths.

Indian

16 chataks = 1 katha

20 kathas = 1 Bigha

One bigha is 1600 sq. yards ;

3 bigha and 8 chataks make one acre.

1936 bighas make one square mile.

The students should particularly note that

1 rasi or bigha (linear) × 1 rasi or bigha (linear) = 1 bigha (area)

1 rasi or bigha (linear) × 1 katha (linear) = 1 katha (area)

1 rasi or bigha (linear) × 1 chatak (linear) = 1 chatak (area)

1 katha (linear) × 1 katha (linear) = $\frac{4}{5}$ chataks (area)

1 katha (linear) × 1 chatak (linear) = $\frac{1}{20}$ chatak (area)

1 chatak (linear) × 1 chatak (linear) = $\frac{1}{320}$ chatak (area)

The country method of land measure is different in different parts of the country. The following tables give a few specimens :

Pakhi (Assam)

15 haths* = 1 nal

6 nals × 5 nals = 1 pakhi

16 pakhis = 1 khada

Kani (East Bengal)

12 haths* = 1 nal

12 nal × 10 nal = 1 kani

16 kanis = 1 drone

$\frac{1}{20}$ of a kani = 1 ganda

$\frac{1}{4}$ ganda = 1 kara

(in some places 24 nals × 20 nals = one kani.)

* This length is variable.

UNITED PROVINCES

20 anubanshis = 1 kachbanshi
20 kachbanshis = 1 biswanshi
20 biswanshis = 1 biswa
20 biswas = 1 bigha
1 bigha = 3025 sq. yd.

MADRAS

24 main or 100 gooli = 1 kani = 6400 sq. yd.

BOMBAY

1 kathi (9·4 ft. × 9·4 ft.) = 88·36 sq. ft.
20 kathis = 1 pund
10 punds = 1 bigha
6 bighas = 1 rookah
20 rookahs = 1 chahar.

The students should carefully note that an area of 2 square inches is quite a different thing from a 2-inch square.

An area of 2 square inches is the space covered by two one-inch squares, while a 2-inch square contains (2 × 2 =) 4 one-inch squares.

Illustrated Examples

Ex. 1. A rectangular field is 15 yd. 1 ft. long and 9 yd. 2 ft. broad. Find its area.

Here

the length = 15 yd. 1 ft. = 46 ft.
the breadth = 9 yd. 2 ft. = 29 ft.
∴ the area = 46 × 29 sq. ft.
= 1334 sq. ft.
= 148 sq. yd. 2 sq. ft.

Ex. 2. The side of a square field measures 5 chains 34 links. Find its area in acres and decimals. Express also the result in acres, roods, poles, etc.

$$\begin{aligned} \text{the area} &= 5{\cdot}34 \times 5{\cdot}34 \text{ sq. chains} \\ &= 28{\cdot}5156 \text{ sq. chains} \\ &= 2{\cdot}85156 \text{ acres} \\ &= 2 \text{ acres, } 3 \text{ roods, and } 16{\cdot}2496 \text{ poles.} \end{aligned}$$

Ex. 3. A rectangular field measures 5 bighas (or rasis) 17 kathas long, and 3 bighas (or rasis) 12 kathas broad. Find its area.

Here the length = 5 bighas 17 kathas = $5\frac{17}{20}$ bighas

the breadth = 3 bighas 12 kathas = $3\frac{12}{20}$ bighas

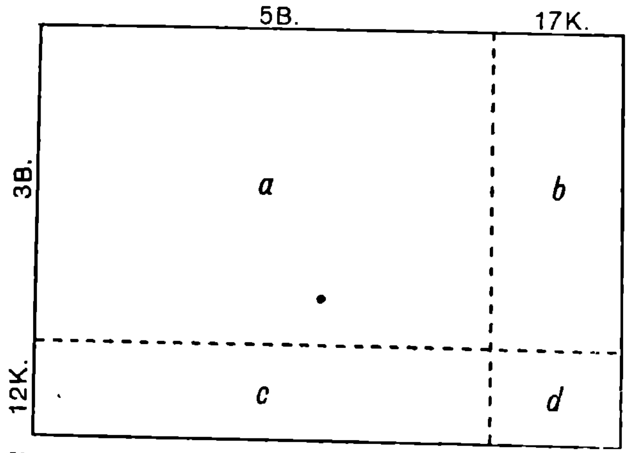
$\therefore$ the area = $5\frac{17}{20} \times 3\frac{12}{20}$ bighas

$$= \frac{117}{\underset{5}{\cancel{20}}} \times \frac{\overset{9}{\overset{\cancel{18}}{\cancel{72}}}}{\underset{10}{\cancel{20}}} = \frac{1053}{50}$$

$= 21\frac{3}{50}$

= 21 bigha 1 katha $3\frac{1}{5}$ chk.

The above can be worked out in the following manner.



In the diagram

area of a = 5 b. × 3 b. = 15 b.

area of b = 3 b. × 17 k. = 51 k. = 2 b. 11 k.

area of c = 5 b. × 12 k. = 60 k. = 3 b.

area of d = 17 k. × 12 k. = 204 × $\frac{4}{5}$ ch. = $163\frac{1}{5}$ ch.

= 10 k. $3\frac{1}{5}$ ch.*

* Please refer to table of Square Measure in Article 26 (*supra*).

∴ The area of the rectangle :

$$\begin{array}{rrrr} & 15 \text{ b.} & & \\ + & 2 \text{ b.} & 11 \text{ k.} & \\ + & 3 \text{ b.} & & \\ + & & 10 \text{ k.} & 3\frac{1}{5} \text{ ch.} \\ \hline = & 21 \text{ b.} & 1 \text{ k.} & 3\frac{1}{5} \text{ ch.} \end{array}$$

Ex. 4. The area of a square field is 4·225 acres. Find the length of the side in chains.

Here the area = 4·225 acres = 42·25 sq. chains.

Again the area = side^2

$$\therefore \text{ side} = \sqrt{\text{area}}$$
$$= \sqrt{42{\cdot}25 \text{ sq. chains.}}$$
$$= 6{\cdot}5 \text{ chains.}$$

Ex. 5. A two-acre field is 80 yd. wide. Find its length.

$$2 \text{ acres} = 2 \times 4840 \text{ sq. yd.}$$

$$\text{length} = 2 \times \frac{4840}{80} = 121 \text{ yd.}$$

Ex. 6. The sides of a rectangular field are in the ratio of 4 to 3, and the area is 2700 sq. yd. Find the side.

Let x = length

then $\frac{3x}{4}$ yd. = breadth.

$$\therefore \text{ Area} = x \times \frac{3x}{4} = \frac{3x^2}{4} \text{ sq. yd.}$$
$$= 2700 \text{ sq. yd.}$$

$$\therefore x^2 = 2700 \times \frac{4}{3} = 3600 \text{ sq. yd.}$$

$$x = 60 \text{ yd.}$$

The length = 60 yd.

The breadth = 45 yd.

Ex. 7. The diagonal of a square field measures 6 chains 48 links. Find its area.

$$\text{Area} = \text{side}^2$$

$$\text{Diagonal}^2 = 2\ \text{side}^2$$

$$= 2\ \text{area}.$$

$$\therefore \frac{\text{Diagonal}^2}{2} = \text{area}.$$

$$\text{Here diagonal} = 6{\cdot}48\ \text{chains}.$$

$$\text{Area} = \frac{6{\cdot}48^2}{2}$$

$$= 20{\cdot}9952\ \text{sq. chains}$$

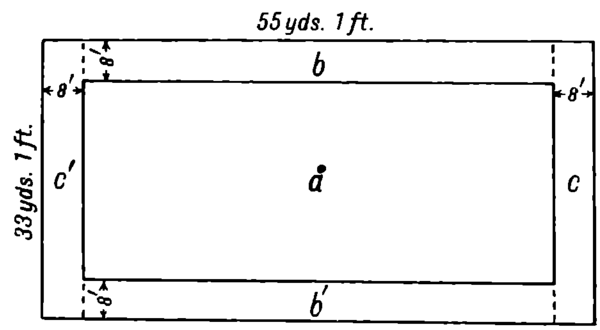
$$= 2{\cdot}09952\ \text{acres}.$$

Ex. 8. A court-yard which is 55 yd. 1 ft. in length and 33 yd. 1 ft. in breadth, contains a rectangular lawn surrounded by a gravel path 8 ft. wide. (*a*) Find the cost of turfing the lawn at the rate of Rs.–/8/– per 100 sq. ft. (*b*) Find the cost of tarmacadamizing the path at the rate of Rs. 2/– per 100 sq. ft.

Here

$$\text{Length} = 55\ \text{yd.}\ 1\ \text{ft.} = 166\ \text{ft.}$$

$$\text{Breadth} = 33\ \text{yd.}\ 1\ \text{ft.} = 100\ \text{ft.}$$



Length of the lawn

$$= 166-(8+8)$$

$$= 150\ \text{ft.}$$

$$\text{Breadth} = 100-(8+8)$$

$$= 84\ \text{ft.}$$

Area $= 150 \times 84 = 12600$ sq. ft.

Cost of turfing the lawn $= \frac{12600}{100} \times \frac{1}{2} =$ Rs. 63.

∴ Area of the path $= b + b' + c + c'$

$= 2b + 2c$

$= 2(150 \times 8) + 2(100 \times 8)$

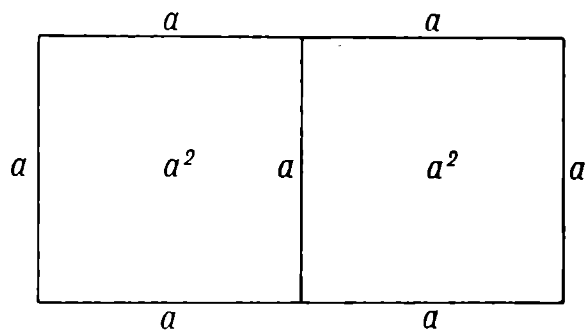
$= 2 \times 8(150 + 100)$

$= 4000$ sq. ft.

Cost of tarmacadamizing the path $= \frac{4000}{100} \times 2 =$ Rs. 80.

Ex. 9. The area of a rectangle is 648 sq. ft. and one side is double of the other.
Find the sides.

Let breadth $= a$;
then length $= 2a$
Area $= 2a^2$;
or $a^2 = \frac{648}{2} = 324$
$\therefore a = 18$ ft.;
breadth $= 18$ ft.
length $= 36$ ft.



Ex. 10. The area of a rectangular field is 1·5 acres and its diagonal is 6·5 chains. Find the length and breadth.

Let $a =$ length, $b =$ breadth.

Then $a^2 + b^2 = \text{diagonal}^2 = (6{\cdot}5)^2 = 42{\cdot}25$ sq. chains.

Area $= ab = 1{\cdot}5$ acres $= 15$ sq. chains.

$(a+b)^2 = a^2 + b^2 + 2ab$

$= 42{\cdot}25 + (2 \times 15)$

$= 72{\cdot}25$

$\therefore a + b = \sqrt{72{\cdot}25} = 8{\cdot}5$ chains.

again,

$$(a-b)^2 = a^2+b^2-2ab$$
$$= 42{\cdot}25-(2\times 15)$$
$$= 12{\cdot}25$$

$$\therefore\ a-b = \sqrt{12{\cdot}25}$$
$$= 3{\cdot}5$$

$$a+b = 8{\cdot}5$$
$$a-b = 3{\cdot}5$$
$$2a = 12$$

$$\therefore\ a = 6 \text{ chains}$$
$$\text{and } b = 2{\cdot}5 \text{ chains.}$$

Ex. 11. The area of a rectangular field 60 chains in perimeter is 10 acres less than the area of a square field with the same perimeter. Find the dimensions of the rectangle.

Let a = length, b = breadth.

The side of the square is $\frac{60}{4}$ chains or 15 chains.

$\therefore$ the area of the square $= 15^2 = 225$ sq. chains.
$= 22{\cdot}5$ acres.

Area of the rectangle $= ab = (22{\cdot}5-10) = 12{\cdot}5$ acres.
$= 125$ sq. chains.

$$a+b = \text{half perimeter} = 30 \text{ chains.}$$
$$(a+b)^2 = a^2+b^2+2ab$$
$$(a-b)^2 = a^2+b^2-2ab$$
$$\therefore\ (a-b)^2 = (a+b)^2-4ab$$
$$= 30^2-4\times 125$$
$$= 900-500$$

$$\therefore\ a-b = \sqrt{400}$$
$$= 20 \text{ chains.}$$

$$a+b = 30 \text{ chains}$$
$$a-b = 20 \text{ chains}$$
$$2a = 50 \text{ chains.}$$

$$\therefore\ a = 25 \text{ chains}$$
$$\text{and } b = 5 \text{ chains.}$$

Ex. 12. A path of uniform width runs round the interior of a courtyard 148 ft. long by 112 ft. broad and encloses a lawn whose area is 13600 sq. ft. Find the width of the path.

See the figure in Ex. 8 above.

Let the path be x ft. wide.

Then the area of the lawn

$$= (148-2x)\times(112-2x)$$
$$= 13600 \text{ sq. ft.}$$

$$\text{or } 16576+4x^2-520x = 13600 \text{ sq. ft.}$$

$$\text{or } 16576-13600+4x^2-520x = 0$$

dividing both sides by 4,

$$744-x^2-130x = 0$$

$$\text{or } (x-6)\ (x-124) = 0$$

$\therefore$ either $x = 6$, or, $x = 124$

The latter result cannot be accepted as the path cannot be wider than the courtyard itself.

$\therefore$ the breadth of the path = 6 ft.

Exercise 3

1. Find the areas of the following rectangles :
   (i) Length 5 yd. 2 ft., breadth 2 yd. 1 ft.
   (ii) Length 73 ft., breadth 37 ft.
   (iii) Length 6 yd. 2 ft. 9 in., breadth 4 yd. 1 ft. 5 in.
   (iv) Length 5·64 chains, breadth 3·42 chains.
   (v) Length 4 bighas 17 kathas, breadth 3 bighas 11 kathas.
   (vi) Length 8 bighas 13 kathas 8 chataks, breadth 6 bighas 9 kathas 12 chataks.

2. Find the area of the following rectangles, giving result in acres :
   (i) Length 23 chains, breadth 11 chains.
   (ii) Length 62 chains, breadth 24 chains.
   (iii) Length 5·68 chains, breadth 4·32 chains.
   (iv) Length 11 chains 42 links, breadth 8 chains 76 links.
   (v) Length 1340 links, breadth 872 links.
   (vi) Length 4 chains 20 links, breadth 82 yd. 1 ft. 6 in.
   (vii) Length 214 ft. 6 in., breadth 148 ft. 6 in.

3. Find the area of the following squares :

(i) Side, 5 ft. 4 in.
(ii) Side, 825 yd. Give the result in acres.
(iii) Side, 5 chains 73 links. Give the result in acres.
(iv) Side, 8·62 chains. Give the result in acres.
(v) Side 4 bighas 11 kathas 8 chataks.
(vi) Side 14 bighas 6 kathas 10 chataks.
(vii) Diagonal 682 yd. Give the result in acres.
(viii) Diagonal 13·46 chains. Give the result in acres.
(ix) Diagonal 35 ft. 4 in.
(x) Diagonal 10 chains 42 links. Give the result in acres.

4. Find the length of the rectangles having the following :

(i) Area 5 sq. yd., breadth 1 yd. 2 ft.
(ii) Area 1 rood, breadth 20 yd.
(iii) Area 1·5 acres, breadth 90 yd.
(iv) Area 3 acres 3 roods, breadth 3 chains 75 links.
(v) Area 59 sq. yd. 1 sq. ft. 2 sq. in., breadth 6 yd. 1 ft. 7 in.
(vi) Area 18 bighas 4 kathas $3\frac{1}{2}$ chataks, breadth 3 bighas 4 kathas 12 chataks.

5. Find the breadth of the rectangles having the following :

(i) Area 2·4 acres, length 16 chains.
(ii) Area 4 acres, length 110 yd.
(iii) Area 6·25 acres, length 1000 links.
(iv) Area 15 bighas 10 kathas 8 chataks, length 5 bighas 3 kathas 8 chataks.

6. Find the side of the squares having the following :

(i) Area 463 sq. yd. 3 sq. ft. 125 sq. in.
(ii) Area 54·756 acres. Give the result in chains and links.
(iii) Area 15 acres 1 rood 20·16 poles. Give the result in chains and links.
(iv) Area 20 acres. Give the result in yd. ft. and in.
(v) Area 8 acres 3 roods 25 poles. Give the result in yd. and ft., etc.

7. Find the diagonals of the squares having the following :

(i) Area 23 sq. yd. 6 sq. ft. 80 sq. in.
(ii) Area 7·28 acres. Give the result in chains and links.

(iii) Area 52 acres 1 rood 36 poles. Give the result in chains and links.

(iv) Area 4·46 acres. Give the result in yd. ft. and in.

(v) Area 17 bighas 8 kathas and $1\frac{3}{5}$ chataks.

8. A rectangular field is 17 chains in length, and it is one-fourth as wide as it is long. What is its acreage ?

9. Find the cost of paving a rectangular area, 40 yd. long and 36 yd. broad, at the rate of 3 annas per sq. ft.

10. What is the area in square feet of the wall of a room 25 ft. long, 15 ft. wide and 14 ft. high ?

11. Find the cost of painting the ceiling of a hall 24 ft. 7 in. long and 20 ft. 10 in. wide, at the rate of 1*s.* 6*d.* per sq. yd.

12. Find the cost of carpeting a room 16 ft. 8 in. long and 14 ft. 2 in. wide at the rate of Rs. 3 per sq. yd.

13. What length of matting 2 ft. wide is required to cover the floor of a room whose area is 44 sq. yd. ?

14. The perimeter of a square is 13 chains 64 links. Find its area in acres.

15. A tennis court is 44 yd. long and 27 yd. 1 ft. 6 in. wide. Find the cost of sowing it with grass seed at the rate of Rs. 16–8–0 per acre.

16. Find the cost of erecting a fencing round a square whose area is 641 sq. yd. 7 sq. ft., at the rate of Rs. 2–8–0 per foot.

17. The cost of tiling a floor is Rs. 492 at the rate of 12 annas per sq. ft., and the breadth of the floor is 8 yd. Find the length.

18. If the cost of a fence round a square plot is Rs. 416 at the rate of Rs. 3–4–0 per yd., what is the rent of the plot at the rate of Rs. 6–4–0 per acre ?

19. How long will a man take to walk round a square field containing 40 acres at the rate of four miles an hour ?

20. The length of a rectangular field is four times its breadth. If the rent of the field is Rs. 250 at the rate of Rs. 6–4–0 per acre, what will be the cost to put up a hedge round it at the rate of Rs. 5–8–0 per 100 ft. ?

21. The perimeter of a rectangular field is 402 yd., and the area is 9860 sq. yd. Find the sides.

22. The area of a rectangular field is 6 acres, and its diagonal is 13 chains. Find its length and breadth in chains.

23. The area of a rectangular field is 4·2 acres, and its diagonal is 12 chains 50 links. How long would it take to walk round the field at the rate of 4 miles per hour ?

24. A square and a rectangle each have a perimeter of 180 chains. The difference between the areas of two figures is 2·5 acres. Find the length and breadth of the rectangle.

25. How many yards of paper 27 in. wide are required for a wall 30 ft. 10 in. long and 15 ft. high ?

26. How many tiles 12 in. long and 6 in. wide will be required to pave a rectangular area which measures 21 ft. by 16 ft. ?

27. If 64 planks, 6 ft. 3 in. long, 9 in. wide, are required to floor a room 12 ft. wide, what is its length ?

28. Find the cost of carpeting a room 26 ft. long by 21 ft. broad with carpet 1 ft. 6 in. wide, at Rs. 4 a yard.

29. Find the cost of paving a courtyard 38 ft. long and 31 ft. wide, with bricks 9 in. long and 4 in. wide at the rate of Rs. 5 for 100 bricks.

30. The side of a rectangular area is 72 ft., and its diagonal is 97 ft. Find the cost of paving it at the rate of 5 annas a sq. ft.

31. How many feet of planking 14 in. wide will be required for the floor of a room 23 ft. 4 in. long and 19 ft. 3 in. wide ?

32. If 70 yd. of carpet is required for a room 31 ft. 6 in. long, by 22 ft. 6 in. wide, what is the width of the carpet in inches ?

33. A courtyard 72 ft. long by 56 ft. broad contains a rectangular lawn surrounded by a path 6 ft. wide. Find the cost of turfing the lawn at the rate of Rs. 2 per 100 sq. ft., and of metalling the road at Rs. 20 per 100 sq. ft.

34. A room is 23 ft. 6 in. long and 19 ft. 3 in. wide. What must be the area of a carpet which leaves a uniform margin of floor 2 ft. 6 in. wide?

35. A carpet 26 ft. 4 in. long and 22 ft. 4 in. wide when placed in a room 30 ft. long leaves a uniform margin. Find the area of the room in sq. ft.

36. A courtyard 80 ft. long and 64 ft. broad contains a rectangular lawn surrounded by a path of uniform width. If the area of the lawn is 3780 sq. ft., find the width of the path.

37. A path of uniform width runs round the interior of a quadrangle 49 ft. 6 in. long and 37 ft. broad, and encloses a lawn whose area is 126 sq. yd. Find the cost of gravel for the path at the rate of 9 annas a sq. yd.

38. A room is 20 ft. in length and 16 ft. in breadth. Allowing $\frac{1}{3}$ of the area for doors and windows it costs Rs. 28 to paper the walls at the rate of 7 annas a sq. yd. What is the height of the room?

39. The seating area of a cinema hall is 69 ft. 6 in. long and 40 ft wide. How many people will it accommodate if it is calculated that each person will require 6 sq. ft.?

40. A hall 15 ft. wide is paved with tiles 9 in. square. How many rows of tiles will there be? How many tiles must be bought to cover the hall if it is 20 ft. long, assuming that a tile cannot be cut without wasting the part cut off?

41. A lawn-tennis court 126 ft. long by 60 ft. wide has a walk of uniform width round and inside it. The area of the walk is $\frac{1}{6}$ of the area of the lawn inside the walk. Find the width of the walk.

42. I have two rectangular sheets of paper, one 1 ft. 2 in. by 1 ft. 8 in., and the other 1 ft. 3 in. by 1 ft. 11 in. I cut them up in a suitable manner and fit the pieces together (without waste) and form a square. Find the length of a side of the square.

43. If 72 cwt. of lead will cover a roof 48 ft. long and 32 ft. wide with lead $\frac{1}{12}$ of an inch thick, what is the length of a roof 25 ft. wide which requires 75 cwt. of lead $\frac{1}{8}$ of an inch thick, to cover it?

44. A square field of grass containing 10 acres is to be cut by machine, working round and round and starting from a corner, which cuts a width of 6 ft. How many rounds must be taken to cut $\frac{9}{25}$ of the field? And how many more to cut another $\frac{7}{25}$?

45. I have a rectangular field $\frac{3}{4}$ of an acre to fence round which costs Rs. 35 at the rate of 2 annas a yard. Find the lengths of the sides.

Would it be possible to have a field of $\frac{3}{4}$ of an acre to fence round which would cost Rs. 30 at the above rate? Give reasons for your answer.

46. It is sometimes said that an inch of rain corresponds to 100 tons of water per acre. Find the error in this statement having given that 1 cubic foot of water weighs $62\frac{1}{2}$ lb.

Examination Questions

47. A room whose length is 30 ft. and breadth twice its height takes 144 yd. of paper 2 ft. wide for its four walls. Find the area of the floor.

48. A rectangular field of 5 acres, 200 yd. long is planted with trees in rows perpendicular to the length, one yard from row to row, and one yard from tree to tree in the same row. If a width of a yard all round the field remain unplanted, find the number of trees.

49. Find the cost of lining a rectangular cistern 12 ft. 9 in. long, 8 ft. 3 in. broad, and 6 ft. 6 in. deep, with sheet lead weighing 8 lb. per sq. ft., and which costs £1 8*s*. per cwt.

50. Find in square feet the total area of the walls, floor, and ceiling of a room $22\frac{1}{2}$ ft. long, $16\frac{1}{2}$ ft. wide, and $13\frac{1}{3}$ ft. high.

51. A square field contains 31 acres 0 roods 10·25 sq. poles. Find the length of a side.

52. A rectangular grass plot, the sides of which are 2 : 3, costs £14 8*s*. for turfing, at the rate of 4*d*. per sq. yd. Find the lengths of its sides.

53. The area of the two side walls of a rectangular room is 806 sq. ft.; the area of the two end walls is 546 sq. ft. Find the dimensions of the room.

54. Two square fields jointly contain 6 acres, and the side of one is three-fourths as long as that of the other. How many acres in each ?

55. There is a garden 140 ft. long and 120 ft. broad, and a gravel walk is to be made of an equal width all round it, so as to take up just one-fourth of the garden. What must be the breadth of the walk ?

56. The breadth of a rectangular room is two-thirds its length, and it costs £39 8*s*. 8*d*. to cover the floor with carpet 27 in. wide, at 5*s*. 3*d*. a yard. To paper the walls costs £2 3*s*. 4*d*. with paper 21 in. wide at 2*s*. 4*d*. per piece of 12 yd. Find the height of the room.

57. In the centre of a room which is 19 ft. $1\frac{1}{2}$ in. square, a square carpet is placed, the rest of the floor being covered by a parquet border of uniform width, which is charged for at the rate of $7\frac{1}{8}$*d*. per sq. ft. If the carpet is charged for at $10\frac{1}{8}$*d*. per sq. ft. and the

whole cost of carpet and parquet is £14 13*s.* $3\frac{3}{4}d.$, find the width of the parquet border.

58. There are two rectangular rooms of the same height, one is 19 ft. by 14 ft., the other 17 ft. by 15 ft. To cover the walls with paper 27 in. wide at 3*s.* 9*d.* per piece of 12 yd. costs £3 12*s.* $2\frac{2}{3}d.$ Find the height of the rooms.

59. Two square rooms, one 2 ft. longer each way than the other, are of equal heights, and cost respectively £3 14*s.* 9*d.* and £3 8*s.* 3*d.* to paper the walls at $6\frac{1}{2}d.$ per sq. yd. Find the height.

60. The cost of a square field at £2 14*s.* 6*d.* per acre, amounts to £27 5*s.* Find the cost of putting a paling round the field at 9*d.* per yd.

61. The length of a room is double the breadth; the cost of colouring the ceiling, at $4\frac{1}{2}d.$ per sq. yd., is £2 12*s.* 1*d.*, and the cost of painting the four walls at 2*s.* 4*d.* per sq. yd. is £35. Find the height of the room.

62. The area of a rectangular courtyard is 2000 sq. yds., and its sides are in the ratio of 1·25 to 1. A pavement of uniform width runs along the four sides of the courtyard, and occupies half its area. Find the width of the pavement.

63. A box, without a lid, and made of wood 1 in. thick requires painting inside and out; its exterior length, breadth, and depth are 3, 2, and $1\frac{1}{2}$ ft. respectively; how many superficial feet of paint will be required for each coat?

64. In a rectangular garden 120 ft. long and 90 ft. broad, a walk passing round it, with its outer edge 10 ft. from the wall occupies a fourth part of the garden; what is the width of the walk?

65. A square field of grain, containing 10 acres is to be cut by a reaper working round and round. The cut of the reaper is 5 ft. How many rounds must the reaper take to cut three-fourths of the field?

66. A rectangular enclosure is 120 ft. long and 70 ft. broad; a walk of uniform width is made round the outside of it equal in area to the enclosure. Find the width of the walk.

67. Find what length of wall paper, 27 in. wide, will be required for a room 20 ft. long, 16 ft. broad, and $10\frac{1}{2}$ ft. high. In it are two windows 6 ft. by 4 ft., a door 7 ft. by 4 ft., and a fireplace 4 ft. by 3 ft. 6 in.

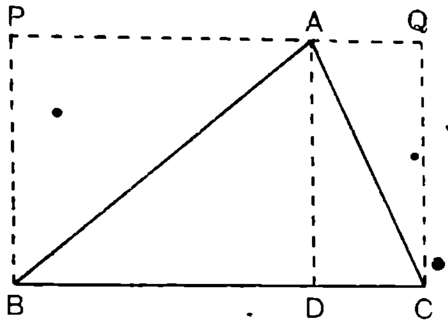
68. A garden is 160 ft. long and 120 ft. broad; there is a tank in the garden, leaving a space of equal width round it, and occupying half the area of the garden. Find the length and breadth of the tank.

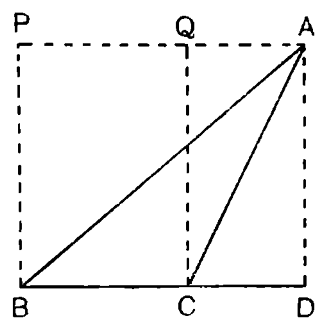
69. The area of a rectangular field is 15 acres, and its length is half as much again as its breadth. How long will it take a man to walk four times round it at the rate of 3 miles an hour?

70. A room 18 ft. long, 15 ft. wide, and 12 ft. high, contains two doors 7 ft. by 4 ft., and (4 ft. from the ground) two windows 4 ft. by 3 ft.; a dado $2\frac{1}{2}$ ft. high runs round the room. Find the cost of papering the walls at 1 anna per sq. ft.

CHAPTER VII

AREA OF TRIANGLES





27. Let ABC be a triangle.

Draw AD perpendicular to BC (Fig. 33) or BC produced (Fig. 34). Through A draw a line parallel to BC. Draw BP and CQ parallel to AD.

Since the diagonal AB bisects the rectangle APBD, the area of the triangle ABD is half the area of rectangle APBD ; and since the diagonal AC bisects the rectangle ADCQ the area of the triangle ADC is half the area of the rectangle ADCQ.

Or triangle ADB $= \frac{1}{2}$ rectangle APBD (a)

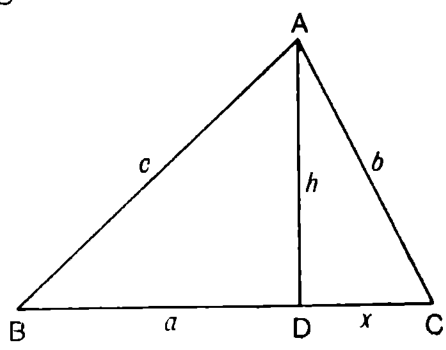
and triangle ADC $= \frac{1}{2}$ rectangle ADCQ (b)

Adding (a) and (b) for Fig. 33 and subtracting (b) from (a) for Fig. 34 we have

$$\begin{aligned} \text{triangle ABC} &= \tfrac{1}{2} \text{ rectangle PBCQ} \\ &= \tfrac{1}{2}\ \text{BC} \times \text{AD} \\ &= \tfrac{1}{2}\ \text{base} \times \text{height}. \end{aligned}$$

That is to say that the area of a triangle is equal to half the product of the base and the height.

We may take any side as "base" remembering that the height or altitude means the perpendicular distance of that side from the opposite angular point.



28. It is also possible to calculate the area of a triangle if the lengths of three sides are known. In the above figure, let BC = a, AC = b, AB = c, AD = h, DC = x

$$\text{then } BD = a-x$$

$$h^2 = c^2-(a-x)^2 = b^2-x^2$$

$$\therefore\ c^2-a^2-x^2+2ax = b^2-x^2$$

$$\text{Or } c^2-a^2+2ax = b^2$$

$$\text{Or } 2ax = b^2+a^2-c^2$$

$$\text{Or } x = \frac{b^2+a^2-c^2}{2a}$$

$$\text{Also } h^2 = b^2-x^2 = (b-x)\ (b+x)$$

$$= \frac{2ab-a^2-b^2+c^2}{2a} \times \frac{2ab+a^2+b^2-c^2}{2a}$$

$$= \frac{(c+a-b)\ (c-a+b)\ (a+b+c)\ (a+b-c)}{4a^2}$$

$$\text{Area of triangle} = \tfrac{1}{2}\, ah$$

$$= \tfrac{1}{2}a \frac{\sqrt{(c+a-b)\ (c-a+b)\ (a+b+c)\ (a+b-c)}}{2a}$$

$$= \tfrac{1}{4}\sqrt{(c+a-b)\ (c-a+b)\ (a+b+c)\ (a+b-c)}$$

To make it easier to remember the above formula is written as under :

$$\text{Denoting } a+b+c = 2s$$

$$\text{then } a+b-c = 2(s-c)$$

$$a-b+c = 2(s-b)$$

$$b+c-a = 2(s-a)$$

Substituting these values we get,

$$\text{Area of the triangle} = \tfrac{1}{4}\sqrt{2s\times 2(s-a)\times 2(s-b)\times 2(s-c)}$$

$$= \sqrt{s(s-a)\ (s-b)\ (s-c)}^{*}$$

* This result was first proved by Hero (about 120 B.C.) of Alexandria, in Egypt, but he solved the problem in the following manner :

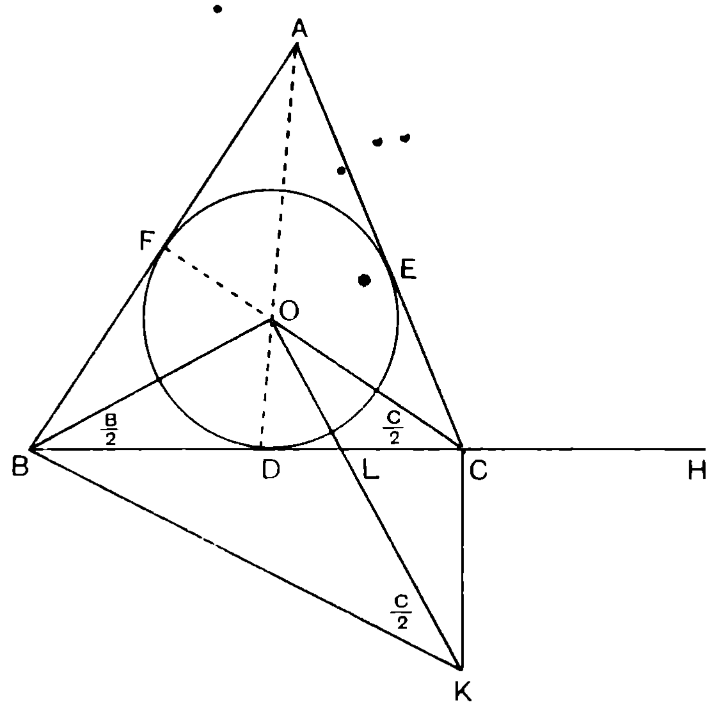
Inscribe the circle DEF.

On BC produced make CH = AF, therefore BH = s.

Draw OK perpendicular to BO and CK perpendicular to BC.
Let r be the radius of the inscribed circle.

Area of triangle ABC =

$$\tfrac{1}{2}\,ar+\tfrac{1}{2}\,br+\tfrac{1}{2}\,cr$$
$$=\tfrac{1}{2}(a+b+c)r$$
$$= sr = BH.\ OD.$$



Now the angles BOK and BCK being right angles, the semicircle on BK will pass through O and C. Therefore the $\angle OCB = \angle OKB$, being angles on the same segment.

The angle OCB = $\tfrac{1}{2}C$

The angles OKB+OBC+CBK + one rt. angle = $\tfrac{1}{2}A+\tfrac{1}{2}B+\tfrac{1}{2}C$

$\therefore\ \angle CBK = \tfrac{1}{2}A = \angle OAF$.

Hence the triangles OAF and CBK are similar.

$\therefore$ BC : CK = AF : OF = CH : OD

$\therefore$ BC : CH = CK : OD = CL : LD

$\therefore$ BH : CH = CD : LD

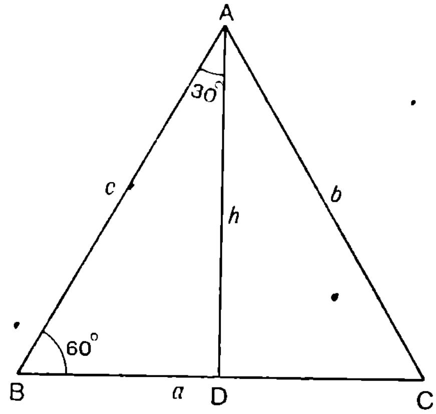
$\therefore$ $BH^2 : CH.BH = CD.BD : LD.BD = CD.BD : OD^2$

$\therefore$ $(BH.OD)^2 = CH.BH.CD.BD$

But area of $\triangle$ = BH.OD

Hence area of $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$.

29. Area of an equilateral triangle :

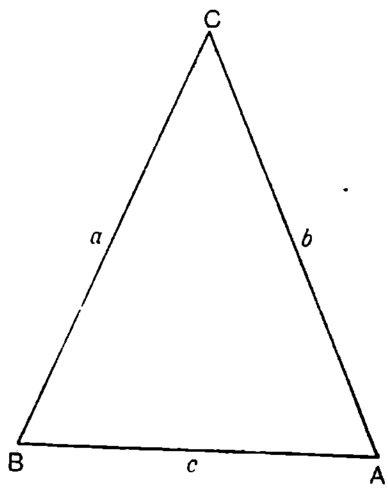


The height or h $= \frac{a}{2}\sqrt{3}$

$$\text{Area} = \frac{a \times a\sqrt{3}}{2 \times 2}$$

$$= \frac{a^2\sqrt{3}}{4}$$

30. Area of an isosceles triangle :



Let $a = b$

$$\therefore \ s = \frac{c+2a}{2}$$

Hence area =

$$\sqrt{\left(\frac{c+2a}{2}\right)\left(\frac{c+2a}{2}-a\right)\left(\frac{c+2a}{2}-a\right)\left(\frac{c+2a}{2}-c\right)}$$

$$= \sqrt{\left(\frac{c+2a}{2}\right)\left(\frac{c}{2}\right)\left(\frac{c}{2}\right)\left(\frac{2a-c}{2}\right)}$$

$$= \frac{c}{4}\sqrt{4a^2-c^2}$$

Illustrated Examples

Ex. 1. The sides of a triangle are 132, 125, 37 yd. respectively. Find the area.

$$s = \frac{132+125+37}{2} = \frac{294}{2} = 147.$$

$$\begin{aligned} \text{area} &= \sqrt{147(147-132)(147-125)(147-37)} \\ &= \sqrt{147\times15\times22\times110} \\ &= \sqrt{7\times7\times3\times5\times3\times11\times2\times11\times2\times5} \\ &= 7\times3\times11\times2\times5 \\ &= 2310 \text{ sq. yd.} \end{aligned}$$

Note: The labour of multiplication and extraction of square root of the product may be avoided partly or wholly by breaking up the numbers into factors.

Ex. 2. Find the area of an equilateral triangle the side of which is 17 ft.

$$\begin{aligned} \text{area} &= 17^2\times\frac{\sqrt{3}}{4} = 289\times\frac{\sqrt{3}}{4} \\ &= 125{\cdot}14 \text{ sq. ft.} \end{aligned}$$

Ex. 3. The area of an isosceles triangle is ·363 acres and the base is 440 links. Find the side.

$$\text{area} = \frac{c}{4}\sqrt{4a^2-c^2}$$

Here area = ·363 acres = 36300 sq. links

and c = 440 links.

$$\therefore \frac{440}{4}\sqrt{4a^2-440^2} = 36300.$$

$$\text{or } \sqrt{4a^2-193600} = \frac{\overset{330}{\cancel{36300}}}{\cancel{110}}$$

$$\text{or } 4a^2 = 330^2+193600$$

(by taking square of each term).

$$\text{or } a^2 = \frac{108900+193600}{4}$$

$$\text{or } a = \sqrt{\frac{302500}{4}} = \frac{550}{2} = 275 \text{ links.}$$

Ex. 4. A lawn is in the form of an isosceles triangle and the cost of turfing it came to Rs. $13\frac{1}{2}$ at the rate of Rs. $4\frac{1}{2}$ per 100 sq. ft. If the base be 40 ft. find the side.

Area of isosceles triangle $= \frac{c}{4}\sqrt{4a^2-c^2}$

$$\text{Here area} = \frac{13\frac{1}{2}}{4\frac{1}{2}} \times 100 \text{ sq. ft.} = 300 \text{ sq. ft.}$$

and $c = 40$ ft.

$$\therefore \frac{40}{4}\sqrt{4a^2-40^2} = 300$$

$$\text{or } \sqrt{4a^2-1600} = \frac{300\times 4}{40}$$

or (taking square of each term) $4a^2-1600 = 30^2$

$$\text{or } 4a^2 = 30^2+1600$$

$$a^2 = \frac{900+1600}{4}$$

$$\therefore a = \sqrt{625} = 25 \text{ ft.}$$

Ex. 5. Find the side of that equilateral triangle whose area costs as much to pave at 10 annas a sq. ft. as it would cost to fence the three sides at Rs. 4 per ft.

Let the side $= x$

$$\text{then the area} = \frac{x^2\sqrt{3}}{4}$$

$$\therefore \text{Cost of paving} = \frac{10\times x^2\sqrt{3}}{4} \text{ annas}$$

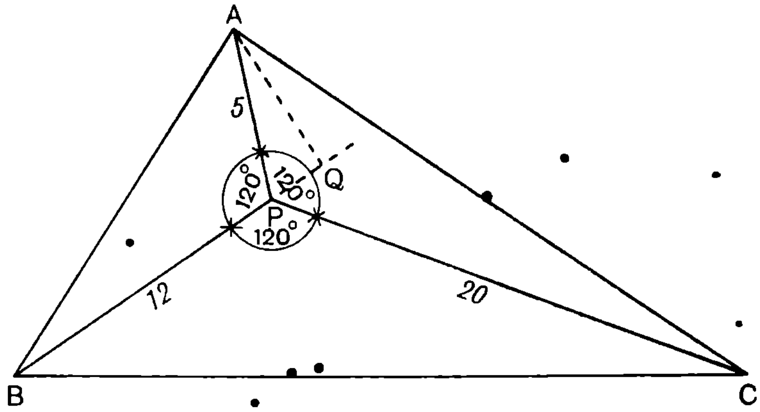
and cost of fencing $= 4\times 3x\times 16$ annas.

But cost of paving = cost of fencing

$$\therefore \frac{10\times x^2\sqrt{3}}{4} = 4\times 3x\times 16$$

$$\text{and } x = \frac{4\times \cancel{3}\times 16\times 4\sqrt{3}}{10\times \cancel{3}} = 44{\cdot}3392 \text{ ft.}$$

Ex. 6. From a point P within a triangle ABC, it is found that the three sides subtend equal angles. If PA, PB, PC measure 5, 12 and 20 chains respectively, find the area of the triangle.



$$\angle APB = \angle APC = \angle BPC$$

$\therefore$ each of these angles $= 120°$

Draw the perpendicular AQ on BP produced.

angle $APQ = 60°$

$$\therefore \; AQ = AP\frac{\sqrt{3}}{2}$$

$$\text{Area of triangle } ABP = \tfrac{1}{2}\, BP.AQ$$
$$= \tfrac{1}{2}\, BP.AP\frac{\sqrt{3}}{2}$$

Similarly

$$\text{Area of triangle } BPC = \tfrac{1}{2}\, PC.BP\frac{\sqrt{3}}{2}$$

$$\text{Area of triangle } APC = \tfrac{1}{2}\, PC.AP\frac{\sqrt{3}}{2}$$

triangle ABC = triangle ABP + triangle BPC + triangle APC.

$$= \left(\tfrac{1}{2}\, 12\times 5\frac{\sqrt{3}}{2}\right) + \left(\tfrac{1}{2}\, 20\times 12\frac{\sqrt{3}}{2}\right) + \left(\tfrac{1}{2}\, 20\times 5\frac{\sqrt{3}}{2}\right)$$

$$= (15+60+25)\sqrt{3}$$

$$= 173{\cdot}2 \text{ sq. chains}$$

$$= 17{\cdot}32 \text{ acres.}$$

Ex. 7. The difference between the area of a square and an equilateral triangle described on the same base is 6·5 acres. Find the length of the common base.

$$\text{Let the base} = a \text{ chains}$$

$$\text{Then area of the square} = a^2$$

$$\text{and area of the triangle} = a^2\frac{\sqrt{3}}{4}$$

$$\therefore\ a^2 - a^2\frac{\sqrt{3}}{4} = 65 \text{ sq. chains}$$

$$\text{or } a^2\left(1 - \frac{\sqrt{3}}{4}\right) = 65$$

$$\text{or } a^2 = \frac{65}{1 - \frac{\sqrt{3}}{4}}$$

$$\text{or } a^2 = \frac{65\left(1 + \frac{\sqrt{3}}{4}\right)}{\frac{13}{16}}$$

Multiplying both numerator and denominator by $1 + \frac{\sqrt{3}}{4}$

$$\text{or } a = \sqrt{114{\cdot}64}$$

$$\therefore\ a = 10{\cdot}7 \text{ chains.}$$

Ex. 8. The area of a right angled triangle is 30 sq. in. and the hypotenuse is 17 in. Find the other sides.

Let the sides be a and b.

$$\text{then the area} = \frac{ab}{2}$$

$$\text{Now } (a+b)^2 = a^2+b^2+2ab$$

$$(a^2+b^2 = \text{hyp.}^2;\ 2ab = 4\times\text{area})$$

$$= 13^2+4\times30$$

$$= 289$$

$$\therefore\ a+b = 17$$

$$\text{again } (a-b)^2 = a^2+b^2-2ab$$

$$= 17^2-4\times30$$

$$= 49$$

$\therefore$ a$-$b $= 7$
and a$+$b $= 17$
$\therefore$ a $= 12$ and b $= 5$.

Ex. 9. The perimeter of a right angled triangle is 40 ft. and its area is 60 sq. ft. Find the sides.

Let the sides adjacent to the right angle be a and b, and the hypotenuse, c.

then the area $= \frac{1}{2}$ ab ; and $c^2 = a^2+b^2$.

Here a+b+c = 40 ft., and $\frac{1}{2}$ ab = 60 sq. ft.

$$c = 40-(a+b)$$
$$\text{or } c^2 = [40-(a+b)]^2$$
$$\text{or } c^2 = 1600+a^2+b^2+2ab-80a-80b$$
$$\text{or } O = 1600+240-80(a+b)$$
$$\text{or } a+b = \frac{1840}{80} = 23$$
$$\therefore\ c = 40-23 = 17$$
$$(a-b)^2 = a^2+b^2-2ab$$
$$= 17^2-(2\times 120)$$
$$= 289-240$$
$$= 49$$
$$\therefore\ a-b = 7,\ a+b = 23$$
$$\therefore\ a = 15 \text{ and } b = 8.$$

The sides are 17, 15 and 8 ft. respectively.

Ex. 10. The perimeter of a triangle is 48 ft. If one side is 10 ft., and the area 84 sq. ft., find the two remaining sides.

The perimeter is 48 ft. $\therefore$ s = 24 ft.

Let the required sides be a and b respectively.

Then

$$\text{area}^2 \text{ or } 84^2 = 24\times(24-10)\,(24-a)\,(24-b)$$
$$\text{or } \frac{84\times 84}{24\times 14} = 576+ab-24a-24b$$
$$\text{or } 24a+24b-ab = 576-21$$
$$\text{or } 24(a+b)-ab = 555$$
$$\text{or } 24(48-10)-ab = 555$$
$$\text{or } 912-555 = ab$$
$$\text{or } 357 = ab.$$

C

$$(a-b)^2 = (a+b)^2 - 4ab$$
$$(a-b)^2 = 38^2 - (4 \times 357)$$

$$\text{or } a-b = \sqrt{1444-1428}$$

$$\text{or } a-b = 4$$

$$\text{and } a+b = 38$$

$\therefore$ a = 21 and b = 17. Sides are 21 ft. and 17 ft.

Exercise 4

1. Find the areas of the right angled triangles, in which the sides adjacent to the right angle are:

   (i) 6 ft. and 13 ft.

   (ii) 15 chains 23 links and 11 chains 76 links. Give the result in acres.

   (iii) 5 rasis 12 kathas, and 4 rasis 17 kathas.

   (iv) 245 yd. and 121 yd. Give the result in acres, roods, poles, etc.

2. Find the area in acres of the right angled triangle in which the hypotenuse is 24 chains 10 links, and one side containing the right angle, 12 chains.

3. Find the hypotenuse in which

   (i) the area is 60 sq. ft., and one side containing the right angle is 15 ft.

   (ii) the area is $2\frac{2}{5}$ acres and one side containing the right angle is 8 chains.

   (iii) the area is 2·1 acres and one side containing the right angle is 77 yd. Give the result in yd.

4. Find the two sides adjacent to the right angle in the right angled triangles having

   (i) the area 7 sq. ft. 72 sq. in., hypotenuse 6 ft. 6 in.

   (ii) the area 1349 sq. yd. 3 sq. ft., hypotenuse 176 yd. 2 ft.

   (iii) the area 12·54 acres, hypotenuse 24 chains 10 links.

5. The perimeter of a right angled triangle is 300 yd. and the area is 3750 sq. yd. Find the sides.

6. The area is 1·33056 acres and the perimeter is 18·48 chains. Find the sides.

7. A field in the form of a right angled triangle has an annual rent of Rs. 26–4–0 at the rate of Rs. 3–2–0 per acre, and to erect a fencing round the field it costs Rs. 495–12–0, at the rate of annas 6 per yard. Find the lengths of sides of the field in yards.

8. The taxes of an allotment in the form of a right angled triangle amount to £19 13*s*. 9*d*. at the rate of 5*s*. 3*d*. an acre. If it takes a man 30 minutes to walk round the area, walking at the rate of $3\frac{3}{4}$ miles per hour, find the lengths of the sides in chains.

9. Find the areas of the following triangles having

(i) base 26 ft., height 17 ft.
(ii) base 4 yd. 1 ft., height 7 yd. 2 ft.
(iii) base 19 yd. 1 ft. 8 in., height 14 yd. 2 ft. 5 in.
(iv) base 14 chains 73 links, height 31 chains 57 links. Give the result in acres.
(v) base 5 rasis 17 kathas, height 6 rasis 12 kathas.

10. Find base or height of the following triangles having

(i) area 132 sq. ft., base 22 ft.
(ii) area 104 sq. yd. 36 sq. in., height 8 yd. 2 ft. 3 in.
(iii) area 17 acres, height 8 chains 30 links.
(iv) area 11·4 acres, base 468 links.

11. Find the length of the perpendicular drawn from the right angle to the hypotenuse of a right angled triangle when

(i) the sides containing the right angle measure 12 ft. and 5 ft.
(ii) the sides containing the right angle measure 48 chains and 14 chains.
(iii) the hypotenuse is 58 ft. and one side 42 ft.

12. Find the areas of the following equilateral triangles having

(i) side 18 ft.
(ii) side 5 yd. 2 ft. 6 in.
(iii) side 12 chains 8 links.
(iv) side 3 rasis.
(v) the perimeter 75 chains.

13. The area of an equilateral triangle is 30 sq. yd. Find the perimeter.

14. The perpendicular drawn from a vertex of an equilateral triangle to the opposite side is 7 chains. Find the perimeter in yards and inches.

15. The perpendicular drawn from a vertex of an equilateral triangle to the opposite side measures 24 chains. Find the area of the triangle in acres.

16. The perimeter of an equilateral triangle measures as many yards as the area of the triangle measures square yards. Find the length of a side.

17. Find the area of the following triangles in which the sides are

(i) 8 yd. 1 ft., 5 yd. 2 ft., 4 yd.

(ii) 51 ft., 37 ft., 20 ft.

(iii) 1250, 850, 600 links. Give the result in acres.

(iv) 5·33, 3·64, 1·95 chains. Give the result in sq. yd.

(v) 2002, 3080, 3234 links. Give the result in acres.

18. The perimeter of an isosceles triangle is 64 ft. and the base is 24 ft. Find the area.

19. The perimeter of an isosceles triangle is 100 ft., and the base is 32 ft. Find the area.

20. Find the equal sides of an isosceles triangle the area of which is 240 sq. ft., and base 20 ft.

21. Find the equal sides of an isosceles triangle whose area is 16·8 acres, and base 14 chains.

22. A plot of ground is in the form of an isosceles triangle. If it costs Rs. 1000 at the rate of Rs. 2–8–0 per square yard, and if each of the equal sides measures 40 yd., find the length of the base.

23. The sides of a triangular field are 25 chains, 17 chains, 28 chains, and its rent is £70. What is the rate of rent per acre ?

24. The sides of a triangle measure respectively 18 yd., 13 yd., 11 yd. Find the length of the perpendicular on the longest side from its opposite vertex.

25. The sides of a triangle are 68 ft., 75 ft., 77 ft. Find to the nearest inch the side of a square equal in area.

26. An isosceles triangle and an equilateral triangle are described on the same base of 16 ft.; area of the first triangle is double that of the other. Find the length of the equal sides.

27. A right angled isosceles triangle is equal in area to an equilateral triangle whose sides are 5 inches long. Find the lengths of its sides.

28. The perimeter of a triangle is 54 ft., and the area is 126 sq. ft. If one side is 13 ft., find the other sides.

29. The perimeter of a triangle is 36 yd., and one side is 17 yd. If the area is 34 sq. yd., find the other sides.

30. The perimeter of a triangle is 54 chains and the area is 9 acres. If one side is 17 chains, find the other sides.

31. The cost of fencing a triangular field is Rs. 371–4–0 at the rate of Rs. 6–4–0 per 100 yd., and the rent of the area is Rs. 737–7–0 at the rate of Rs. 3–2–0 per acre. If one side measures 1320 yd., find the other sides.

32. Two sides of a triangle measure 12·50 chains and 24·50 chains, and include an angle of 60°. Find the side of an equilateral triangle equal in area.

Examination Questions

33. The sides of a triangle are 25, 39, 56 ft. respectively. Find the perpendicular from the opposite angle on the side of 56 ft.

34. The area of a triangle is 336 sq. ft. and the sides are 26 ft. and 30 ft. Find the base.

35. The three sides AB, AC, BC of the triangle ABC are 68, 75 and 77 ft., respectively. Find the length of the perpendicular from A on BC.

36. The area of an equilateral triangle is 25 sq. in. Find its perimeter.

37. Find the least possible length of fencing that can include a triangular area of 10 sq. ft.

38. The sides of a triangle are 13, 14, and 15 ft. Find the perpendicular from the opposite angle on the side of 14 ft.

39. The sides of a triangle are 7, 24 and 25 ft., respectively. Find the area.

40. The sides of a triangle are $2\frac{1}{4}$, 3, $3\frac{3}{4}$ ft. Find in inches the area of the triangle.

41. Two sides of a triangle are 85 and 154 ft., respectively, and the perimeter is 324 ft. Find the area of the triangle.

42. The side of an equilateral triangle is 7 ft. Find the area.

43. The sides of a triangle are 18, 20 and 22 respectively. Calculate its area to three places of decimal.

44. Find the area of a triangle the sides of which are 20, 493 and 507 yd. respectively.

45. The side of an equilateral triangle is 10 ft. Find the area in sq. ft.

46. An equilateral triangle measures 362 sq. ft. Find the length of one side.

47. An equilateral triangle measures 1 acre. Find the length of a side in feet.

48. Determine the area of a triangular plot of ground whose sides are 8 ft., 10 ft., 12 ft.

49. Find the area of a triangular field whose sides are 1200, 1800 and 2400 links. (Answer to be given in acres, roods, and perches.)

50. An acre and a half of land, in the form of a right angled triangle, is divided into two parts by a line which bisects the right angle, and which measure $82\frac{1}{2}$ yd. Find the two areas.

51. Find the area in acres, roods, and perches, of a field whose sides are 848, 900, and 988 links.

52. What is the side of that equilateral triangle whose area costs as much paving at 8*d.* a foot as railing the three sides at a guinea a yard ?

53. The sides of a triangle are 51, 52, 53 ft. Find the perpendicular from the opposite angle on the side of 52 ft., and find the areas of the two triangles into which the original triangle is divided.

54. The sides of a triangle are 15, 14, 13 ft. Find the area in links.

55. The sides of a triangle are 1200, 1450, and 1650 ft. Find the area in sq. yd.

56. The sides of a triangle are 1115, 1750, and 1765 ft. Find the area in acres, roods, and perches.

57. The sides of a triangle are in the proportion of 13, 14, and 15, and the perimeter is 70 yd. Find the area.

58. The sides of a triangle are in the ratio of 13, 14, 15, and the perimeter is 84 yd. Find the perpendiculars from the angular points upon the sides.

59. The sides of a triangular field are 191, 245, and 310 ft. Find the area in acres.

60. Find the area in acres, roods, and perches, of a triangle whose sides measure 405, 378, and 351 ft.

61. The sides of a triangle are 4789, 3742, and 2987 ft. Find the area in yards.

62. The sides of a triangle are in the proportion of 13, 14, 15, and the perimeter is 50 yd. Find the area.

63. The sides of a triangle are 1137, 1259, and 1344 ft. Find the area in acres, roods, and perches.

64. What is the area of a triangle whose sides are 165, 220, and 275 ft.? Find the answer in acres, roods, and perches.

65. The sides of a triangle, of which the perimeter measures 462 ft., are in the ratio of 6, 7, and 8. Find its area.

66. The perimeter of an isosceles triangle is 306 ft., and each of the equal sides is $\frac{5}{8}$ of the base. Find the area.

67. Find the area of an isosceles triangle whose base is 16 ft. long, and sides each 17 ft. long.

68. Find in acres the area of a triangle whose sides are $\frac{121}{3}\sqrt{6}$, $101\sqrt{24}$, $725\sqrt{\frac{2}{3}}$ yd. respectively.

69. One side of a triangular court is 98 ft., and the perpendicular on it from the opposite angle is 63 ft. Required the expense of paving it at Rs. 1–3–0 per sq. yd.

70. The sides of a triangle are 17, 15, and 8 in. respectively. Find the length of the straight line joining the middle point of 17 to the opposite angle.

71. The sides of a triangle are 25, 101, 114. Find the two parts in which the longest side is divided by the perpendicular from the opposite angle.

72. The sides of a triangular field are 350, 440, and 750 yd., the field is let for £26 5*s*. a year. Find at what price per acre the field is let.

73. The sides of a triangle are 35, 39, and 56 ft. respectively. Find areas of the two triangles into which it is divided by the perpendicular from the opposite angle on the largest side.

74. The sides of a triangle are 143, 407, and 440 yd. respectively. Find the rent of the field at £2 3*s*. per acre.

75. The sides BC, CA, AB of the triangle ABC are 13, 12 and 5 respectively, and D is the middle point of BC. Find the area of the triangle ABC, and the length of the line AD.

76. The area of a triangular field is 2 acres 3 chains. The line drawn from the vertex of the same perpendicular to the base measures 13 poles or perches. What is the length of the base line in chains and links ?

77. A house 42 ft. wide has a roof with unequal slopes, the lengths of which are 26 and 40 ft. Find the height of the ridge above the eaves.

78. Given two sides of an obtuse angled triangle which are 20 and 40 poles, find the third side, that the triangle may contain just one acre of land.

79. A man observes the elevation of the top of a tower to be 60°. He then walks a distance of 300 ft., takes a turn of a right angle, and after walking 400 ft. more finds he is on the other side of the tower, opposite to his original position. The elevation of the tower is now found to be 30°. Find the height of the tower.

80. The three sides of a triangle are 800, 500, and 1237 links. By some mistake the third side was also put down as 500 instead of 1237. What error would that mistake occasion in the computed area ?

81. From a point within an equilateral triangle perpendiculars are drawn to the three sides, and are 8, 10, and 12 ft. respectively. Find the side and the area of the triangle.

82. A garden containing 1 acre is in the form of a right angled isosceles triangle. A walk passing round it at 6 ft. from the boundary wall occupies one-fourth of the whole garden. Find the width of the garden.

83. The base of a triangular field is 1210 yd. and the height is 496 yd. ; the field is let for £248 a year. Find at what price per acre the field is let.

84. The side of a square is 100 ft. ; a point is taken inside the square which is distant 60 ft. and 80 ft. respectively from the ends of a side. Find the areas of the four triangles formed by joining the point to the four corners of the square.

85. Find the side of an equilateral triangle whose area is 5 acres. (Give the answer in feet.)

86. A triangular field, whose sides measure 375, 300, and 225 yd., is sold for £8,500. Find the price for acre.

87. A triangular field is let for £5 11*s.* 6½*d.*, at the rate of £12 an acre. One side is 738 links. Find the perpendicular on this side from the opposite angle.

88. The area of an equilateral triangle is 1943·737 sq. ft. Find its side.

89. Find the cost of painting the gable end of a house at 1*s.* 9*d.* per sq. yd., the breadth being 27 ft., the distance of the eaves from the ground 33 ft., and the perpendicular height of the roof 12 ft.

90. The paving of a triangular court came to £100, at 1*s.* 3*d.* per sq. ft.: if one of the sides be 24 yd. long, find the length of the other two equal sides.

91. What must be the side of an equilateral triangle so that its area may be equal to that of a square of which the diagonal is 120 ft.?

92. A triangular field, 363 yd. long and 240 yd. in the perpendicular, produces an income of £36 a year. At how much an acre is it let?

93. A field, whose three sides are equal, cost Rs. 55–6–9 turfing at the rate of 5 annas per 100 sq. ft. Find the length of one of its sides.

94. In a place where land costs £40 an acre, a triangular field was bought for £300, of which one side measured 302 yd. 1 ft. 6 in. What was the height of this triangle in yards?

95. Find (correct to the thousandth part of an inch) the length of one of the equal sides of an isosceles triangle on a base of 14 in., having an area of 92·4 sq. in.

96. If the length of each side of an equilateral triangle were increased by 1 ft., the area would be increased by $\sqrt{3}$ sq. ft. Find the length of each side.

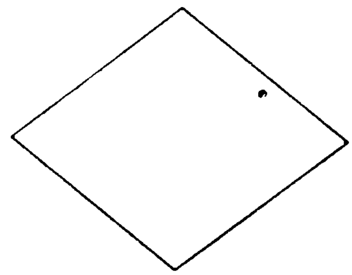
97. A person standing at a point A due south of a tower observes the altitude of the tower to be 60°. He then walks to a point B due west of A, and observes the altitude to be 45°; and again at a point C in AB produced he observes the altitude to be 30°: show that B is midway between A and C.

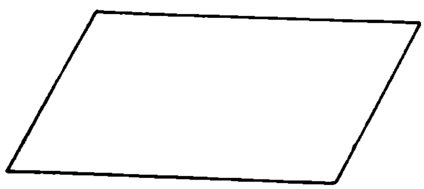
98. The medians of a triangle are 105, 156, 219 ft. respectively. Find the area of the triangle.

CHAPTER VIII

QUADRILATERAL

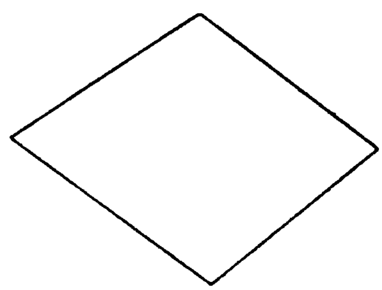
A quadrilateral is a four-sided figure.

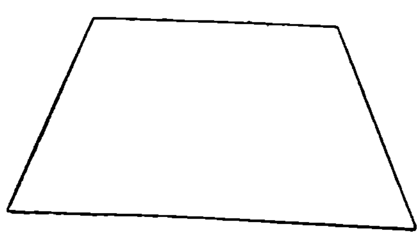




A parallelogram is a quadrilateral whose opposite sides are parallel.

A rhombus is a parallelogram whose sides are all equal and whose angles are not right angles.





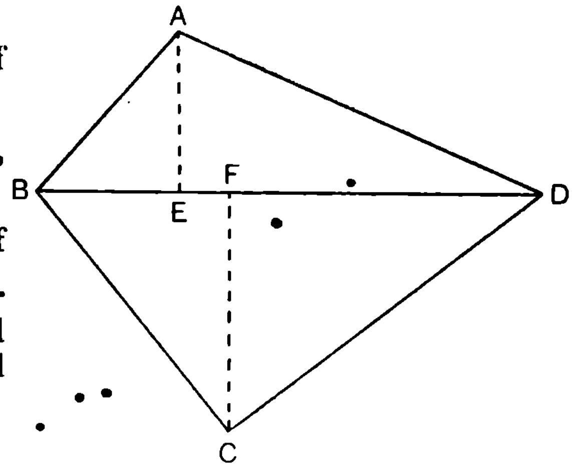
A trapezium or a trapezoid is a quadrilateral having one pair of opposite sides parallel.

31. To find the area of the quadrilateral ABCD.

Case I. Given AB, BC, CD, DA and BD.

Calculate the area of the triangles ABD and BCD.

Case II. Given BD and the perpendiculars CF and AE called offsets.



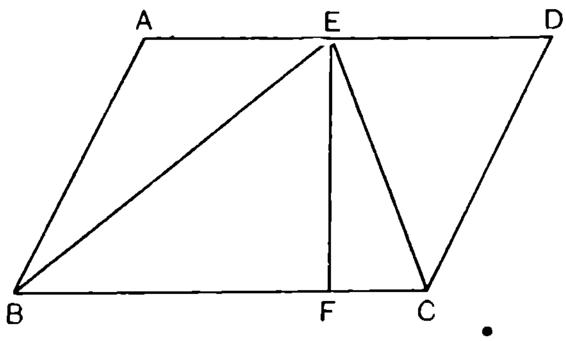
In this case the area of

$$\text{triangle ABD} = \frac{BD.AE}{2}$$

$$\text{triangle BCD} = \frac{BD.CF}{2}$$

Area of the quadrilateral ABCD = triangle ABD + triangle BCD

$$= \tfrac{1}{2}\,BD(AE+CF)$$



32. To find the area of the parallelogram ABCD.

Let EF be a perpendicular on BC; EF is called the height of the parallelogram.

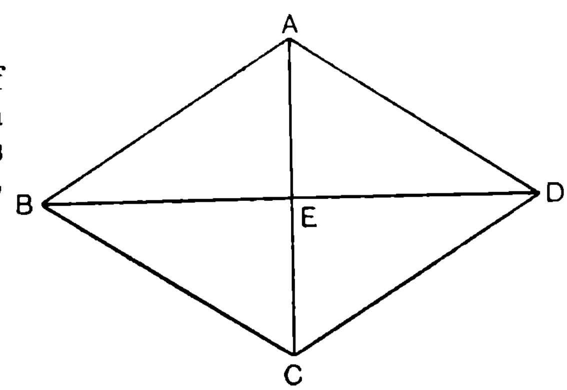
Join BE and CE. Now, area of the parallelogram ABCD $= 2 \times$ area of triangle EBC.

$\therefore$ Area of ABCD $= 2 \times \frac{1}{2}$ BC $\times$ EF = BC $\times$ EF.

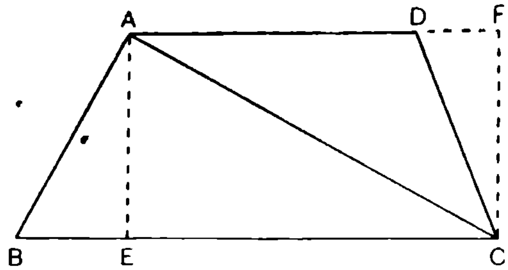
33. To find the area of the rhombus ABCD. In a rhombus the diagonals bisect one another at right angles.

Hence the area of

$$ABCD = \frac{DB \times AC}{2}$$



34. To find the area of the trapezium ABCD.



Draw the perpendiculars AE on BC and CF on AD produced.

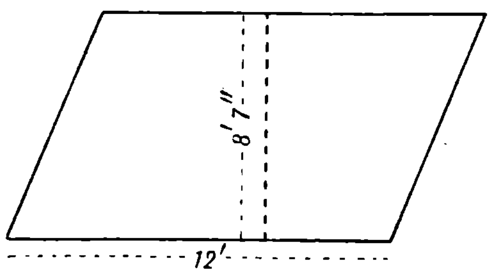
$$AE = CF$$

Now, area of ABCD = area of triangle ABC + triangle ACD

$= \frac{1}{2} BC \times AE + \frac{1}{2} AD \times AE$

$= \frac{1}{2} AE(BC + AD)$

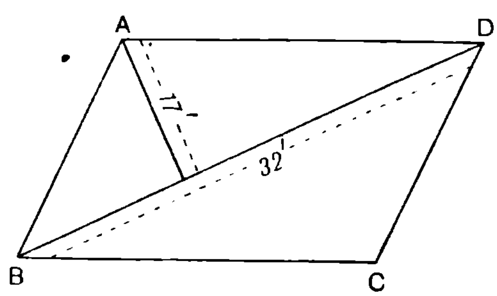
Illustrated Examples



Ex. 1. The base of a parallelogram is 12 ft. and the height is 8 ft. 7 in. Find its area.

The area of the parallelogram = 12 ft. × 8 ft. 7 in. = 103 sq. ft.

Ex. 2. One diagonal of a parallelogram is 32 ft. and the perpendicular drawn from one of the other corners is 17 ft. Find the area of the parallelogram.



The parallelogram is bisected by the diagonal, hence triangle ABD = BCD

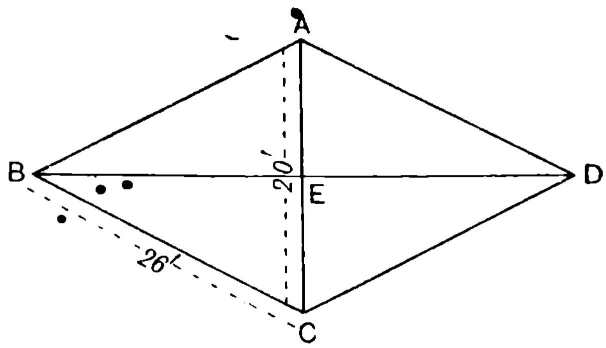
∴ the area of ABCD = 2 area of triangle AB

$= 2 \times \frac{17 \times 32}{2}$

= 544 sq. ft.

Ex. 3. The side of a rhombus is 26 ft. and one of its diagonals is 20 ft. Find the other diagonal and the area of the rhombus.

In the diagram the angle BEC is a right angle, hence

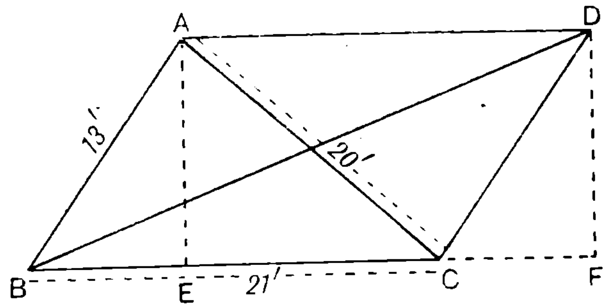


$BE = \sqrt{BC^2 - EC^2}$
$= \sqrt{26^2 - 10^2}$
$= \sqrt{576}$
$= 24$

and BD
$= 2\,BE$
$= 48$ ft.

The area
$= \frac{1}{2} \times 48 \times 20$
$= 480$ sq. ft.

Ex. 4. The sides of a parallelogram are 21 ft. and 13 ft.; one diagonal is 20 ft. Find the other diagonal and the area of the parallelogram.



In the parallelogram ABCD,

AB = 13 ft.,
BC = 21 ft.,
AC = 20 ft.

From A and D draw the perpendiculars AE and DF on BC and BC produced, respectively.

Now $AC^2 = AB^2 + BC^2 - 2BC.BE$ . . (Euc. II–13.)

Again $DB^2 = BC^2 + DC^2 + 2BC.CF$ . . (Euc. II–12.)

Therefore, adding, we get:

$AC^2 + DB^2 = 2(AB^2 + BC^2)$ (since $BE = CF$)

Here

$20^2 + DB^2 = 2(13^2 + 21^2)$
or $DB^2 = 2(169 + 441) - 400$
$= 1220 - 400$
or $DB = \sqrt{820}$
$= 28{\cdot}635$ ft.

Area of ABCD = 2 area of triangle ABC

$= 2\sqrt{s(s-a)(s-b)(s-c)}$

Here $s = \dfrac{13+21+20}{2}$

$= 27$

Hence the area of ABCD $= 2\sqrt{27(27-13)(27-21)(27-20)}$

$= 2\sqrt{27\times14\times6\times7}$

$= 2\times3\times3\times2\times7$

$= 252$ sq. ft.

Ex. 5. The parallel sides of a trapezium are 234 yd. and 368 yd. respectively and the perpendicular distance between them is 193 yd. Find the area of the trapezium.

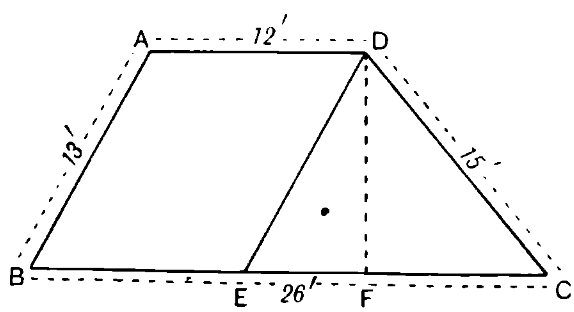
Area $= \dfrac{368+234}{2}\times193$

$= 58093$ sq. yd.

Ex. 6. The parallel sides of a trapezium are 12 and 26 ft., and the other sides are 13 and 15 ft. Find the area.

In the trapezium ABCD

AD = 12 ft., AB = 13 ft., BC = 26 ft., DC = 15 ft.



Through D draw DE parallel to AB, and DF perpendicular to BC.

Then EC = (26−12) ft. = 14 ft.

Now, area of the triangle DEC $=\sqrt{s(s-a)(s-b)(s-c)}$

Where $a = 13$

$b = 14$

$c = 15$

and ∴ $s = 21$

$\therefore$ area of triangle DEC $= \sqrt{21 \times 8 \times 7 \times 6}$

$= 7 \times 3 \times 4$

$= 84$ sq. ft.

Also $DF = \dfrac{2 \text{ area of DEC}}{EC}$

$= \dfrac{2 \times 84}{14}$

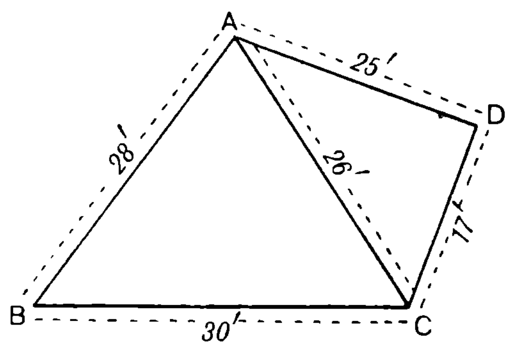
$= 12$ ft.

$\therefore$ Area of trapezium ABCD $= \dfrac{12+26}{2} \times 12$

$= 228$ sq. ft.

Ex. 7. In the quadrilateral ABCD the following measurements are given. Find the area.

AB = 28 ft.,
BC = 30 ft.,
CD = 17 ft.,
DA = 25 ft.;
and the diagonal
AC = 26 ft.



Here the area of the triangle ABC $= \sqrt{42 \times 12 \times 16 \times 14}$

$= \sqrt{14^2 \times 6^2 \times 4^2}$

$= 336$ sq. ft.

and the area of the triangle ACD $= \sqrt{34 \times 17 \times 9 \times 8}$

$= \sqrt{17^2 \times 4^2 \times 3^2}$

$= 17 \times 4 \times 3$

$= 204$ sq. ft.

Area of the quadrilateral ABCD $= 336$ sq. ft. $+ 204$ sp. ft.

$= 540$ sq. ft.

Ex. 8. Find the area of a quadrilateral ABCD in which the diagonal AC measures 5 chains and the perpendicular on it from B and D are 3 chains 40 links, and 4 chains 20 links.

Here the area $= \frac{1}{2} \times 5(3{\cdot}40 + 4{\cdot}20)$

$= \frac{1}{2} \times 5 \times 7{\cdot}60$

$= 19{\cdot}00$ sq. chains

$= 1{\cdot}9$ acres.

Ex. 9. The sides of a quadrilateral inscribed in a circle are 64, 58, 40, and 52 in. Find its area.

The area of a quadrilateral inscribed in a circle

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where the sides are a, b, c, and d

$$\text{and } s = \tfrac{1}{2}(a+b+c+d)^{*}$$

$$\text{Here } s = \tfrac{1}{2}(64+58+40+52) = 107$$

$$\therefore \text{ area} = \sqrt{43\times 49\times 67\times 55}$$

$$= 7\sqrt{43\times 67\times 55}$$

$$= 2786{\cdot}448 \text{ sq. in.}$$

Exercise 5

1. Find the areas of the following parallelograms:

(i) Base 25 ft., height 14 ft.

(ii) Base 11 yd., height 8 yd. 2 ft.

(iii) Base 14·56 chains, height 8·72 chains. Give the area in acres.

(iv) Base 8 chains 15 links, height 5 chains. Give the area in acres.

(v) Base 968 yd., height 605 yd. Give the area in acres.

(vi) Base 5 rasis 12 kathas, height 3 rasis 8 kathas.

(vii) One diagonal 8 ft. 5 in.; perpendicular distance of this diagonal from either of the outlying corners, 3 ft. 2 in.

(viii) One diagonal 10 chains 24 links; perpendicular distance of this diagonal from either of the outlying corners 7 chains 54 links. Give the area in acres.

(ix) Two adjacent sides are 13 ft. and 15 ft. and one diagonal is 16 ft.

(x) Two adjacent sides are 5 chains 60 links, and 8 chains 24 links, and one diagonal is 7 chains 48 links. Give the area in acres.

(xi) The diagonals are 20 chains and 16 chains; one side is 12 chains. Give the area in acres.

* This formula for the area of a cyclic quadrilateral was discovered by Brahma Gupta, who lived about A.D. 600. The proof of this formula depends upon the fact that the opposite angles of a quadrilateral inscribed in a circle are together equal to two right angles (Euc. III–22). The proof is not given here as it involves trigonometry.

2. The area of a parallelogram is 1·75 acres, and each of the two parallel sides measures 2 chains 10 links. Find to the nearest link the perpendicular distance between them.

3. Two adjacent sides of a parallelogram are 231 ft. and 120 ft., and the perpendicular distance between the pair of shorter sides is 77 ft. Find the distance between the other pair.

4. In a parallelogram the perpendiculars between the two pairs of parallel sides are 130 ft. and 182 ft. If the longer side is 238 ft. find the adjacent side.

5. Find the side of a rhombus whose diagonals measure 7 yd. and 5 yd. 1 ft. 6 in. respectively.

6. Find the areas and sides of the rhombuses whose diagonals are :

(i) 4 ft. and 1 ft. 2 in.
(ii) 15 chains and 11 chains.
(iii) 4 rasis 12 kathas and 6 rasis 8 kathas.
(iv) 16 yd. 2 ft. and 13 yd.

7. Find the area of a rhombus on a side of 10 in., one diagonal being also 10 in.

8. A field is in the form of a rhombus whose diagonals are 28·70 chains and 18·50 chains. Find to the nearest penny the rent at £4 10*s*. 6*d*. per acre.

9. Find the area of a rhombus whose perimeter is 82 yd. 2 ft., and one diagonal, 34 yd. 2 ft.

10. The side of a rhombus is 1·30 chains, and one of its diagonals is 2·24 chains. Find the other diagonal.

11. The area of a rhombus is 1080 sq. ft., and the side is 39 ft. Find the diagonals.

12. Find the areas of the following trapezoids :

(i) Parallel sides 83 and 57 ft., perpendicular distance 16 ft.
(ii) Parallel sides 6 ft. 2 in. and 7 ft. 4 in., perpendicular distance 4 ft.
(iii) Parallel sides 4 chains 80 links and 6 chains 46 links ; perpendicular distance 3 chains 24 links.
(iv) Parallel sides 5 rasis 4 kathas and 4 rasis 16 kathas ; perpendicular distance 8 rasis 12 kathas.

13. Find the perpendicular distance between the parallel sides of the trapezoids having the following measurements:

(i) Area 7 acres, parallel sides 8·25 chains, and 5·75 chains.
(ii) Area 340 sq. yd., parallel sides 30 ft. and 18 ft.
(iii) Area 62 acres 3 roods, parallel sides 30 chains and 20 chains.

14. If land costs Rs. 400 an acre, find the cost of a field in the form of a trapezoid whose parallel sides measure 1169 links and 851 links; the perpendicular distance between them being 350 links.

15. The two parallel sides of a trapezium are 58 yd. and 42 yd. respectively; the other sides are equal, each being 17 yd. Find the area.

16. A room is in the form of a trapezium whose parallel sides measure 35 ft. 7 in. and 24 ft. 5 in. respectively. The perpendicular distance between them is 18 ft. What length of matting $\frac{3}{4}$ yd. wide will be required to cover the floor ?

17. The two parallel sides of a field in the shape of a trapezium measure 9 chains 1 link and 13 chains 35 links. If the rent at £2 an acre is £55 18*s*., find the distance between the parallel sides.

18. What will be the cost of paving a courtyard in the form of a trapezoid whose parallel sides measure 20 yd. 2 ft. and 17 yd. 1 ft. respectively, and the perpendicular distance between them 10 yd., at 4 annas per sq. ft. ?

19. In the trapezium ABCD the angles at A and D are right angles and the angle BCD is 120°. If DC = 40 ft., CB = 20 ft., find the area in sq. ft.

20. Find the area of the quadrilaterals whose dimensions are as follows:

(i) Diagonal 18 in., offsets 11 in. and 9 in.
(ii) Diagonal 215 ft., offsets 83 ft. and 117 ft.
(iii) Diagonal 16 yd. 2 ft., offsets 10 yd. 1 ft. and 7 yd. 2 ft.
(iv) Diagonal 12 chains 15 links, offsets 7 chains 56 links and 8 chains 44 links.
(v) Diagonal 22 chains 46 links, offsets 15 chains 28 links and 11 chains 94 links.

21. Find the cost of a quadrilateral piece of ground one of whose diagonals measures 7 chains 40 links, and the offsets 4 chains 26 links and 6 chains 44 links respectively, at Rs. 400 per acre.

22. The difference between the two parallel sides of a trapezoid is 8 ft., the perpendicular distance between them is 24 ft., and the area is 312 sq. ft. Find the two parallel sides.

23. Find the area of a quadrilateral whose diagonals are 5 yd. 2 ft. and 6 yd. 1 ft. respectively, and are at right angles to one another.

24. Find the area of the quadrilateral ABCD in which AB = 13 in., BC = 20 in., CD = 17 in., AD = 10 in., and the diagonal AC = 21 in.

25. In the quadrilateral ABCD the measurements are AB = 26 chains, BC = 17 chains, CD = 17 chains, DA = 12 chains, AC = 25 chains. Find the area in acres.

26. The diagonals of a quadrilateral are at right angles to each other and measure respectively 16 chains 25 links and 24 chains 80 links. Find the area in acres.

27. Find the area of a quadrilateral inscribed in a circle whose sides measure 36, 77, 75, and 40 ft. respectively.

28. Find the area of the quadrilateral ABCD in which the angles at B and D are right angles, and AB = 15 ft., BC = 20 ft., CD = 7 ft.

29. Find the area of the quadrilateral ABCD in which AB = 35 ft. BC = 12 ft., CD = 20 ft., DA = 51 ft., and the angle ABC is a right angle.

30. Find the acreage of the quadrilateral ABCD in which the angle ABC is 60°, the angle ADC is a right angle ; AB = 13 chains, BC = 13 chains, CD = 12 chains.

Examination Questions

31. The diagonals of a rhombus are 6 ft. and 8 ft. Find the side and the height.

32. The diagonals of a rhombus are 72 and 96. Find its area and the lengths of its sides.

33. Each side of a rhombus is 330 ft., and one diagonal is 500 ft. Find the area of the rhombus in acres and cents.

34. Find the area of a rhombus in square feet, the diagonals being 160 ft. and 100 ft.

35. The area of a mat in the form of a rhombus is 8 sq. yd., and the perimeter is 36 ft. Find its perpendicular breadth.

36. The semi-diagonals of a rhombus are 8 in. and 16 in. respectively. Find the area of the rhombus, and also the length of its side.

37. The area of a rhombus is 120,000 sq. ft., and the side 400. Find the diagonals.

38. The diagonals of a rhombus are respectively 40 and 60 yd. Find its area, perimeter, and height.

39. The diagonals of a rhombus are 88 and 234 ft. respectively. Find the area. Find also the length of a side and the height of the rhombus.

40. The side of a rhombus is 36 ft. and one of its diagonals is 18 ft. Find the other diagonal and the area of the figure.

41. The side of a rhombus is 20, and its longer diagonal is 34·64. Find the area and the other diagonal.

42. The area of a rhombus is 354,144 sq. ft., and one diagonal is 672 ft. Find the other diagonal. Find also the length of a side, and the height of the rhombus.

43. The diagonals of a rhombus are 60 ft. and 45 ft. respectively. Find its area. Find also the length of a side and the height of the rhombus.

44. The side of a rhombus is 20 ft., and its shorter diagonal is three-fourths the longer one. Find its area.

45. The diagonals of a rhombus are 4 ft. and 1 ft. 2 in. Find the sides and the area.

46. A field is in the form of a rhombus whose diagonals are 2870 links and 1850 links. Find to the nearest penny the rent at £4 10*s*. 6*d*. an acre.

47. The diagonals of a rhombus are 80 and 60 ft. respectively. Find the area, length of side, and height of the rhombus.

48. The sides of a quadrilateral inscribed in a circle, taken in order, are 25, 39, 60, and 52 ft. Find the area of the quadrilateral.

49. The opposite sides of a quadrilateral are parallel, and the distance between them is 7 chains 50 links. If the area is 6·75

acres, and the length of one of the parallel sides is 10 chains 30 links, find the length of the other.

50. ABCD is a quadrilateral. Each of the angles ABC and DAC is a right angle, the following lengths are in feet: AB = 112, CD = 175, and DA = 105. Find the area.

51. The sides of a quadrilateral, taken in order, are 5, 12, 14 and 15 ft. respectively, and the angle contained by the first two is a right angle. Find the area.

52. In a quadrilateral ABCD, AC = 13 ft., BD = 12 ft., AC cuts BD at right angles. Find the area.

53. The sides of a quadrilateral are 75, 75, 100, 100 ft. respectively, and it can be inscribed in a circle. Find its area.

54. In a trapezoid the parallel sides are 14 and 20 yd. respectively, and the perpendicular distance between them is 12 yd. Find the area of the trapezoid.

55. Find the area of a trapezoid whose parallel sides are 1000 ft. and 1500 ft., and the distance between them 100 ft.

56. A field is in shape a trapezoid, whose parallel sides are 6 chains 75 links and 9 chains 25 links. If the area be 2 acres 3 roods 8 perches, find the shortest way across the field in yards.

57. A field in the form of a quadrilateral, ABCD, whose sides taken in order are respectively equal to 192, 576, 288, and 480 ft., has the diagonal AC equal to 672 ft. Find the area in acres, roods, poles, etc.

58. One diagonal of a quadrilateral which falls without the figure is equal to 30 yd., and the difference of the perpendiculars upon it from the remaining angles of the quadrilateral is 14 yd. Find its area.

59. Find, in acres, the area of a quadrilateral whose diameter is 19·3 chains, and the perpendiculars on which from the opposite angles are 13·5 chains and 18·75 chains respectively. (1 chain = 66 ft.)

60. Find the area in acres of a field ABCD. AD = 220 yd., BC = 265 yd., AC = 378 yd., and the perpendiculars from D and B meet the diagonal in E and F, so that AE = 100 and CF = 70 yd.

61. AC is the diameter of a circle and a diagonal of the inscribed quadrilateral ABCD; given AB = 30, BC = 40, CD = 10, find AD and the area of the quadrilateral.

62. How many square yards are there in a trapezoid, the parallel sides of which are 157·6 metres and 94 metres, and the perpendicular distance between them 72 metres? (1 metre = 39·37 in.)

63. The area of a trapezoid is 475 sq. ft., the perpendicular distance between the two parallel sides is 19 ft. Find the two parallel sides, their difference being 4 ft.

64. Calculate the area of a trapezoid the sides of which, taken in order, are 13, 11, 15, and 25, and the second parallel to the fourth.

65. How many square yards of paving are there in a quadrangular court whose diagonal is 54 ft. and the perpendiculars on it, from the opposite corners, 25 and $17\frac{3}{4}$ ft. respectively?

66. A trapezoid, with parallel sides of lengths as 3 : 4 is cut from a rectangle 12 ft. × 2 ft., so as to have an area of one-third of the latter. Find the lengths of the parallel sides.

67. The parallel sides of a trapezoid are 55 and 77 ft., and the other sides are 25 and 31 ft. Find the area.

68. Two of the four hedges of a field are parallel, and 1000 yd. and 936 yd. respectively. A man standing midway between these parallel hedges observed that a horse he was lunging, with 25 yd. of rope, in crossing the shortest line from his station to either parallel hedge, bisected it. Required, area of field in acres.

69. A quadrilateral ABCD. Find area in acres and decimals of an acre. AB = 300 yd., BC = 350 yd., CD = 700 yd., DA = 650 yd., AC = 400 yd.

70. The sides of a quadrilateral, taken in order, are, 8, 8, 7, 5 ft. respectively, and the angle contained by the first two sides is 60°. Find the area.

71. The parallel sides of a trapezoid are 14 and 30 ft. respectively, and the other two sides are 12 and 19 ft. Find the area.

72. One diagonal of a quadrilateral which lies outside the figure is 70 ft., and the difference of the perpendiculars upon it is 16 ft. Find the area.

73. A railway platform has two of its opposite sides parallel, and its other two sides equal; the parallel sides are 100 and 120 ft. respectively, and the equal sides are 15 ft. each. Find its area.

74. In a trapezium ABCD, AB = 345, BC = 156, CD = 323, DA = 192, the diagonal AC = 438. Find the area.

75. Required the depth of a ditch, the transverse section of which is a trapezoid, area 146·25, breadth at top = 20, side slopes 3 to 1 and 2 to 1.

76. The area of a trapezoid is $3\frac{1}{8}$ acres, the sum of the two parallel sides is 297 yd. Find the perpendicular distance between them.

77. The four sides of a quadrilateral inscribed in a circle are 80, 60, 50, and 86 ft. Required the area.

78. One of the parallel sides of a trapezoid is 1 ft. longer than the other, the breadth is 1 ft., and the area 216 sq. in. Required each of the parallels.

79. How many square yards are contained in a quadrilateral, one of its diagonals being 60 yd. and the perpendiculars upon it 12·6 and 11·4 yd.?

80. Find the area of a trapezoid whose parallel sides are 72 and $38\frac{2}{3}$ ft., the other sides being 20 and $26\frac{2}{3}$ ft.

81. A ditch is 30 ft. wide at top and 18 ft. at bottom. The earth excavated from it is formed into a bank 28 ft. wide at top and 38 ft. at bottom, and 10 ft. high. What is the depth of the ditch.

82. The area of a trapezoidal field is $4\frac{1}{2}$ acres, the perpendicular distance between the parallel sides is 120 yd., and one of the parallel sides is 10 chains. Find the other.

83. ABCD is a quadrilateral, right-angled at B and D; also AB = 36 chains, BC = 77 chains, CD = 68 chains. Find the area.

84. Find an expression for the area of a trapezoid with parallel sides of lengths a and b, and the other sides c and d.

85. Find the area of a quadrilateral ABCD, given AB = 30 in., BC = 17 in., CD = 25 in., DA = 28 in., BD = 26 in.

86. One diagonal of a quadrilateral which falls without the figure is equal to 30 yd., and the difference of the perpendiculars upon it from the remaining angles of the quadrilateral is 40 yd. Find the area.

87. The sides of a quadrilateral are 204, 369, 325, 116 yd., and the second side is parallel to the fourth. Prove that the angle contained by the first two sides is a right angle, and find the area of the quadrilateral.

Chapter IX

POLYGONS

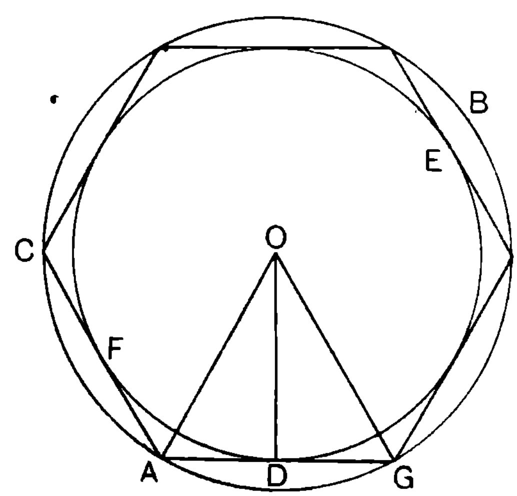
A polygon is a figure bounded by more than four straight lines.

A polygon is said to be regular, in which all the sides and angles are equal.

A polygon	having	five	sides	is called	a	pentagon
,,	,,	six	,,	,,	a	hexagon
,,	,,	seven	,,	,,	a	heptagon
,,	,,	eight	,,	,,	an	octagon
,,	,,	nine	,,	,,	a	nonagon
,,	,,	ten	,,	,,	a	decagon
,,	,,	eleven	,,	,,	an	undecagon
,,	,,	twelve	,,	,,	a	dodecagon
,,	,,	fifteen	,,	,,	a	quindecagon.

35. The area of a quadrilateral is most easily obtained by drawing a diagonal and calculating the areas of the two triangles into which the diagonal divides the quadrilateral. In a similar way the area of any irregular rectilinear figure may be obtained by dividing it into triangles.

36. In the case of a regular polygon it is obvious that the central point of any regular polygon is also the centre of both the circumscribed and inscribed circle. The area of the polygon then consists of a number of equal isosceles triangles having their bases the sides of the polygon and their vertices at the centre of the circumscribed or inscribed circle.



For example O is the centre of the circle ABC which is circumscribed about the polygon as also of the circle DEF which is inscribed in the polygon. OD is the radius of the inscribed circle, and OA is the radius of the circumscribed circle.

OD is perpendicular on AG, hence the area of the triangle OAG = $\frac{1}{2}$ AG.OD.

And the area of the polygon = number of sides in the polygon $\times$ area of the triangle OAG.

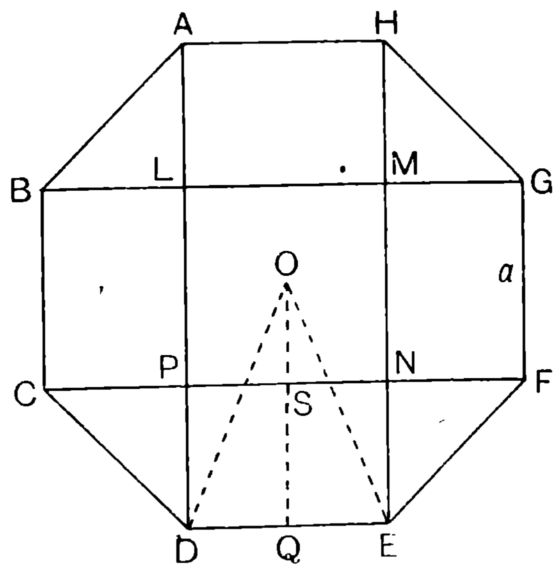
37. Area of a regular hexagon. Here it may be seen that the triangle OAG is equilateral.

$$\therefore \text{ Area of OAG} = \frac{\text{side}}{2}\sqrt{3}\times\frac{\text{side}}{2} = \frac{\text{side}^2\sqrt{3}}{4}$$

and the area of the hexagon $= \dfrac{3\ \text{side}^2\sqrt{3}}{2}$

38. Area of a regular octagon.

Let the side represent a



Here the area = square LPNM+
4 rect. PDEN+4 triangles NEF.

$$= a^2+4a\times\frac{a}{\sqrt{2}}+4\times\frac{1}{2}\frac{a\sqrt{2}}{2}\times\frac{a\sqrt{2}}{2}$$

$$= a^2+2a^2\sqrt{2}+a^2$$

$$= 2a^2+2a^2\sqrt{2} = 2a^2(1+\sqrt{2})$$

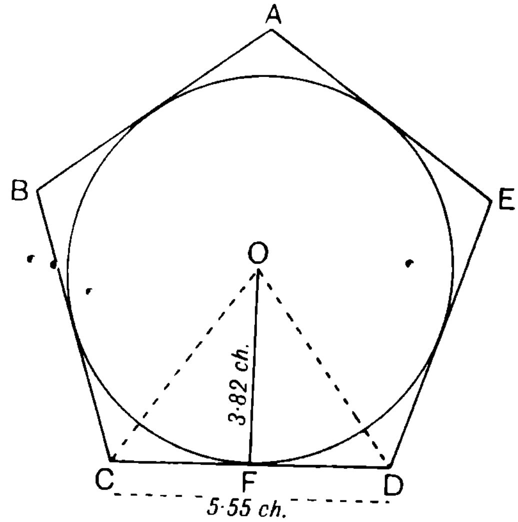
$$= 2a^2\times 2{\cdot}41421$$

$$= a^2\times 4{\cdot}82842.$$

That is to say that the area of a regular octagon is obtained by multiplying the square of the side by 4·82842.

Illustrated Examples

Ex. 1. Find the area of a regular pentagon whose side measures 5 chains 55 links and the radius of the inscribed circle is 3 chains 82 links.



Let ABCDE be the pentagon.

Area of ABCDE = 5 × area of triangle OCD

$= 5 \times \frac{CD \times OE}{2}$

$= 5 \times \frac{5{\cdot}55 \times 3{\cdot}82}{2}$

$= 53{\cdot}0025$ sq. chains

$= 5{\cdot}30025$ acres.

Ex. 2. Find the area of a regular hexagon whose side measures 8 in.

Area of hexagon $= \frac{3\ \text{side}^2\sqrt{3}}{2}$

Here area of the hexagon $= \frac{3 \times 8^2\sqrt{3}}{2}$

$= \frac{3 \times 64\sqrt{3}}{2}$

$= 96\sqrt{3}$

$= 166{\cdot}272$ sq. in.

Ex. 3. Find the area of a regular hexagon inscribed in a circle whose radius is 15 ft.

The radius of the inscribed circle = side of the hexagon.

Here the area $= \frac{3\ \text{radius}^2\sqrt{3}}{2}$

$= \frac{3 \times 15^2\sqrt{3}}{2}$

$= 3 \times 225 \times {\cdot}866$

$= 584{\cdot}55$ sq. ft.

Ex. 4. Find the cost of carpeting an octagonal floor whose side measures 16 ft. at Rs. 2 per sq. yd.

$$\text{Area of octagon} = \text{side}^2 \times 4{\cdot}82842$$

$$\text{Here the cost} = \text{Rs. } \frac{2\times 16\times 16\times 4{\cdot}82842}{9}$$

$$= \text{Rs. } 274{\cdot}88$$

$$= \text{Rs. } 274 \text{ 10 annas 11 pies nearly.}$$

Ex. 5. What must be the side of a regular hexagon of which the area is 1 bigha ?

$$\text{Area of a regular hexagon} = \frac{3 \text{ side}^2\sqrt{3}}{2}$$

$$\therefore \text{ 1 bigha} = \frac{3 \text{ side}^2\sqrt{3}}{2}$$

$$\text{or side}^2 = \frac{1\times 2}{3\sqrt{3}}$$

$$\text{or side} = \sqrt{\frac{2\sqrt{3}}{3\sqrt{3}\times\sqrt{3}}}$$

$$= \sqrt{\frac{2\sqrt{3}}{9}}$$

$$= \frac{\sqrt{3{\cdot}464}}{3}$$

$$= \frac{1{\cdot}86}{3}$$

$$= {\cdot}62 \text{ rasis.}$$

Exercise 6

1. Find the area of the regular pentagon whose side measures 48 ft. 2 in. and the radius of the inscribed circle is 33 ft. 2 in.

2. Find the area of the regular pentagon whose side measures 4 chains 76 links and the radius of the circumscribed circle is 4 chains 5 links.

3. The area of a regular pentagon is $7\frac{1}{2}$ acres and the radius of the inscribed circle is 4 chains 54 links. Find the length of a side.

4. Find the area of a regular hexagon described about a circle whose radius is $7\sqrt{3}$ in.

5. Find the cost of carpeting an octagon floor whose side measures 16 ft., at 4 annas a sq. ft.

Examination Questions

6. An ornamental grass plot is in the shape of a regular hexagon, each side 100 ft.; within the plot and along its sides a footpath is made, 4 ft. wide all round. Find the area of the grass plot left within.

7. Calculate to three decimal places the area of a regular hexagon each of whose sides is equal to 10 ft.

8. Find the area of a regular octagonal field each of whose sides measures 5 chains. (Give the result in acres, roods, etc.)

9. There is a square room which it is proposed to enlarge by throwing out an octagonal front on one of its sides, so that three sides of the octagon may form a bay front. What area of new flooring will be required, the side of the square being 20 ft.?

10. Compare the areas of an equilateral triangle, a square, and a regular hexagon of equal perimeter.

11. The area of a regular octagon is 51 sq. yd. Find the length of its side.

12. The radius of the circumscribed circle of a pentagon is $\sqrt{\frac{3000}{\pi}}$ ft. where $\pi = 3{\cdot}1416$. Find the length of the side, and the area of the pentagon.

13. Find the side of a regular octagon inscribed in a square, the area of which is $6+4\sqrt{2}$ sq. ft.

14. Find the area of a regular hexagon whose perimeter is 3000 ft.

15. Find the area of a hexagon, each side being 30 ft.

16. Find the area of a regular octagon whose side is 20 ft.

17. The front of a room 24 ft. wide is to be projected in the form of three sides of an octagon: construct the projection. How much will it increase the total length of the room in the central line, and how much will it add to the area?

18. The area of a regular octagon is 1086·4 ft. Find the length of one side.

19. Find the areas of a square and a regular hexagon, the perimeter of each being 300 ft.

20. The sides of a five sided figure ABCDE are AB = 25 ft., BC = 29 ft., CD = 39 ft., DE = 42 ft., and EA = 27 ft., also AC = 36 ft., and CE = 45 ft. Find its area.

21. ABCDE is a five-sided figure, and the angles at B, C, and D are right angles; if AB = 20 ft., BC = 18 ft., CD = 32 ft., and DE = 13 ft., find the area of the figure, and the length of AE.

22. The sides of a pentagon taken in order are 100, 130, 197, 133 and 94 ft., and the two diagonals measured from the intersection of the first and last sides are 209 and 193 ft. Find the area of the figure.

23. In the pentagonal field ABCDE, the length of AC is 50 yd. and the perpendiculars from B, D and E upon AC are 10, 20 and 15 yd., the distances from A of the feet of the perpendiculars from D and E being 40 and 10 yd. Find the area.

24. The lengths of the sides (in yards) of a six-sided field ABCDEF are as follows: AB = 31, BC = 130, CD = 38, DE = 41, EF = 130, FA = 22. Given that the angles at A and D are right angles, and that BF is parallel to CE, find the area of the field in square yards.

25. What is the content of the eight-sided figure ABCDEFGH, the diagonal AE being taken as a base, and the perpendiculars drawn from the angular points to AE being Bb, Cc, Dd, etc., and when the lengths of the perpendiculars above the diagonal are Bb = 294, Cc = 142·5, Dd = 224, and those below the diagonal are Ff = 121, Gg = 195·5, Hh = 142, and the intercepted breadths are Ah = 44·5, hb = 124·25, bg = 80, gc = 41, cd = 130·5, df = 50, fE = 52·5 ?

Chapter X

CIRCLE

39. The length of the circumference of a circle bears a fixed ratio to the length of the diameter of the circle.

It is self evident that the perimeter of a polygon circumscribed about a circle is greater than the circumference of the circle and that the perimeter of a polygon inscribed in the circle is less than the circumference of the circle.

By calculating the perimeters of two regular polygons of 96 sides, the one circumscribed and the other inscribed in a given circle, Archimedes* proved that this fixed ratio lay between $3\frac{1}{7}$ and $3\frac{10}{71}$†. The ratio is denoted by the Greek letter π (*pi*), and its value to 5 places of decimals is 3·14159. The ratio cannot be expressed exactly as the ratio of two whole numbers. A common approximation to the value of π is $3\frac{1}{7}$ which is greater than the correct value by less than 0·05 per cent. A still closer approximation is $\frac{355}{113}$ which gives the value accurately to the 5th place of decimals.‡

Thus we have

$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

$$\therefore \text{ circumference} = \pi \times \text{diameter}$$

or if c denotes the length of the circumference, and r the radius, then

$$\frac{c}{2r} = \pi$$

$$\text{and } c = 2\pi r.$$

* Archimedes, the most celebrated of ancient mathematicians, who probably was related to the royal family of Syracuse, in Sicily, was born there in 287 B.C. There was a story, though generally rejected, that Archimedes set fire to a Roman blockading fleet by means of burning glasses and concave mirrors. Archimedes was killed during the sack of the city which followed its capture by the Roman general Marcellus.

† These limits were deduced from the result of his calculation of perimeters of the inscribed and circumscribed polygons which were $\frac{6336}{2017\frac{1}{4}}$ and $\frac{14688}{4673\frac{1}{2}}$

‡ Ingenious calculators have worked out the value of π to more than 700 places of decimals, but the result has no practical value. The following figures give the value to 25 places.

3·14159 26535 89793 23846 26434

This result was verified by Mr. C. G. B. Stevens, I.C.S.

According to the degree of accuracy required in the result the value of π may be taken as $\frac{22}{7}$, 3·1416, $\frac{355}{113}$, or 3·14159.

The Arc of a Circle

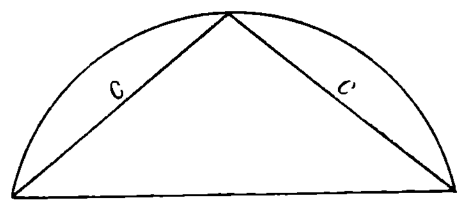
40. In a circle the length of any arc is proportional to the angle which it subtends at the centre of the circle.

This hardly requires proof. If we divide the circumference into 360 equal parts, each part will subtend an angle of 1° at the centre. If an arc subtends an angle of 48° at the centre then the length of that arc $= \dfrac{\text{circumference} \times 48}{360}$.

41. The following rule for calculating the length of the arc of a circle may sometimes be useful.*

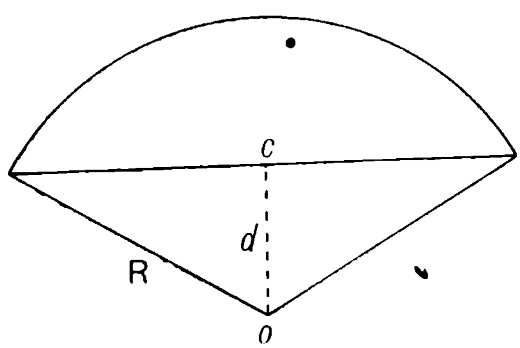
In the Figure, C denotes the chord of the arc and c, the chord of half the arc.

Now, the length of the arc is approximately equal to $\frac{1}{3}(8c-C)$. For small arcs this rule is very accurate. If the arc subtends an angle of 30° at the centre the error is less than 1 in 100,000.



42. Chord of a Circle

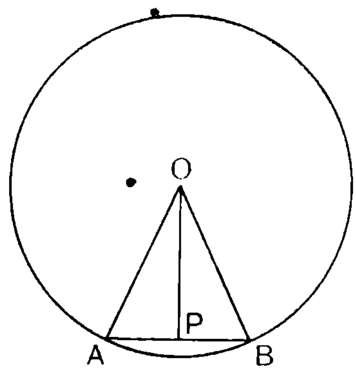
A straight line which bisects the chord of a circle at right angles passes through the centre of the circle. It follows immediately that if R is the radius of the circle, c the chord, and d the distance of the chord from the centre of the circle, then $R^2 = d^2 + \frac{c^2}{4}$.



* This rule is known as Huygens' approximation. Huygens (1629–1695), a Dutch scientist, best known by his improvements in the telescope and his application of the pendulum for regulating clocks. He published this approximation in 1654 in a work called *De Circuli Magnitudine Inventa.* The proof of this approximation is beyond the scope of this work.

43. AREA OF A CIRCLE

Archimedes proved that the area of a circle is equal to the area of a right angled triangle whose sides are equal respectively to the radius and the circumference of the circle ; i.e. the area $= \frac{1}{2}$ radius $\times 2$ radius π.



If the radius be r then the area of a circle $= \pi r^2$.

Let the circumference of the circle be divided into any number (say n) of equal parts.

Suppose AB is one such part. Draw the perpendicular OP.

Area of triangle OAB $= \frac{1}{2}$AB.OP

$\therefore$ Area of the polygon of which AB is one side $= \frac{n}{2}$. AB.OP $= \frac{OP}{2} \times$ perimeter of polygon

(n.AB = perimeter of polygon).

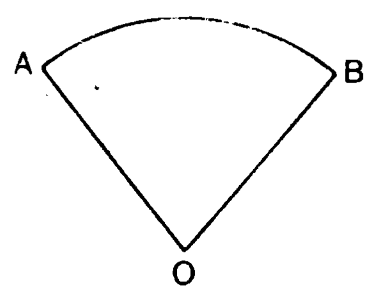
Now, let the number of sides, n, be increased indefinitely. Ultimately, when n is infinitely large, the perimeter of the polygon becomes the circumference of the circle and OP becomes the radius.

$$\therefore \text{ the area of the circle} = \frac{r}{2} \times \text{circumference}$$

$$= \frac{r}{2} \times 2\pi r = \pi r^2$$

44. AREA OF A SECTOR

A sector of a circle is the figure contained by two radii and the arc between them.



In the figure which represents a sector, O is the centre of the circle.

It is evident that the area of the sector is proportional to the angle contained by its radii.

$$\therefore \frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\angle AOB}{360}$$

$$\therefore \text{Area of sector} = \frac{\angle AOB \times \text{Area of circle}}{360}$$

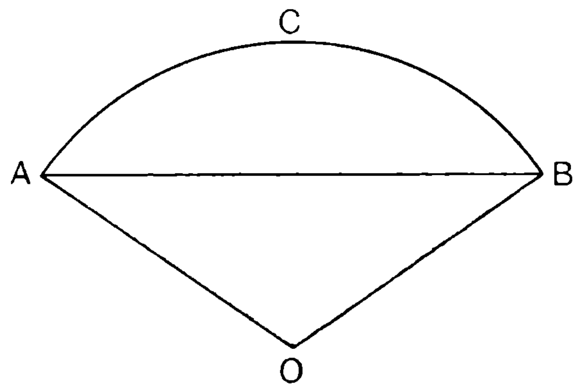
$$= \frac{\angle AOB}{360} \pi r^2$$

$$\text{But length of the arc} = \frac{\angle AOB}{360} 2\pi r$$

$$\therefore \text{Area of sector} = \frac{r}{2} \times \text{length of arc.}$$

45. Segment of a Circle

A segment of a circle is the figure contained by a chord and the arc which cuts it.



Let AB be a chord so that ABC is a segment.

Area of segment ABC = Area of sector AOBC — Area of triangle AOB.

46. Circle Inscribed in a Triangle

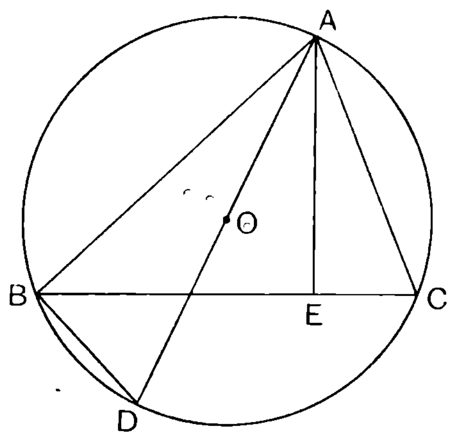
It has been proved, *vide* Art. 28 (footnote*), that the area of a triangle is equal to sr when s is the semi-perimeter and r is the radius of the inscribed circle.

$$\therefore \text{Radius of the inscribed circle} = \frac{\text{Area of triangle}}{\text{semi-perimeter}}$$

47. CIRCLE CIRCUMSCRIBED ABOUT A TRIANGLE

Describe the circle ABDC about the triangle ABC.

Let O be the centre, then, OA is the radius.



Produce AO to meet the circumference at D. Join BD, and draw the perpendicular AE on BC.

$\angle ABD$ is a right angle.

$\angle ACE = \angle BDA$ (being angles on the same segment).

$\therefore$ triangles ABD and AEC are similar.

$\therefore$ AC : AE = AD : AB.

$$\text{But area of triangle} = \tfrac{1}{2}\,BC.\,AE$$

$$\text{or } AE = \frac{2 \text{ triangle}}{BC}$$

$$\therefore\ AC : \frac{2 \text{ triangle}}{BC} = 2\,OA : AB$$

$$\text{or } OA = \frac{AC \times BC \times AB}{4 \text{ triangle}}$$

That is to say that the radius of the circle circumscribed about a triangle is equal to the product of the sides divided by four times the area of the triangle.

Illustrated Examples

Ex. 1. Find the circumference of a circle whose radius is 4 yd. 1 ft. 5 in. ($\pi = \frac{22}{7}$).

Here $r = 4$ yd. 1 ft. 5 in. $= 161$ in.
and circumference $= 2\pi r = 2\times\frac{22}{7}\times 161 = 1012$ in.
$= 28$ yd. 0 ft. 4 in.

Ex. 2. The driving-wheel of a bicycle makes 720 revolutions in travelling a mile, find the diameter of the wheel ($\pi = \frac{22}{7}$).

The circumference of the wheel $= \frac{1760\times 3}{720}$

Diameter $= \frac{1760\times 3\times 7}{720\times 22} = \frac{7}{3}$ft. $= 2$ ft. 4 in.

Ex. 3. The radius of a circle is 18 ft. Find the radius of another circle whose area is one-third of this circle ($\pi = \frac{22}{7}$).

The area of the circle $= 18\times 18\times\frac{22}{7}$.

The area of the 2nd circle $= \frac{18\times 18\times 22}{3\times 7}$

Radius of the 2nd circle $= \sqrt{\frac{18\times 18\times 22\times 7}{3\times 7\times 22}}$

$= \sqrt{6\times 6\times 3}$
$= 6\sqrt{3}$
$= 10{\cdot}392$ ft.

Ex. 4. The height of an arc is 8 ft., and the diameter of the circle is 40 ft. Find the chord of the arc.

$R^2 = d^2+\frac{c^2}{4}$

Here $20^2 = 12^2+\frac{c^2}{4}$

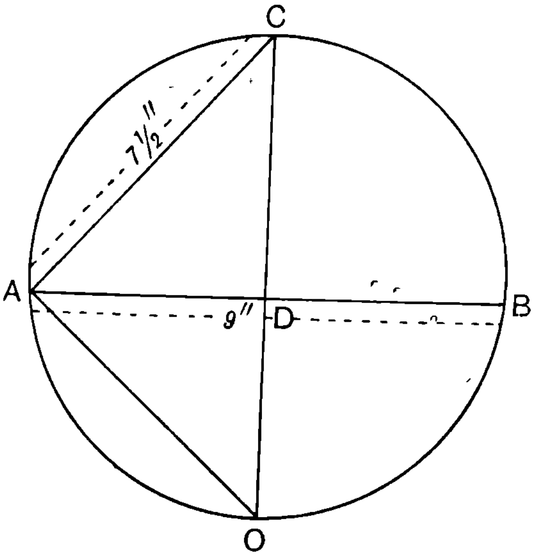
or $20^2-12^2 = \frac{c^2}{4}$

or $256\times 4 = c^2$

or $\sqrt{16\times 16\times 2\times 2} = c$

$\therefore$ $c = 32$ ft.

Ex. 5. The chord of an arc is 9 in. and the chord of half the arc is $7\frac{1}{2}$ in. Find the diameter of the circle.



In the diagram triangles OAC and ACD are similar

$\therefore$ CD : AC = AC : OC

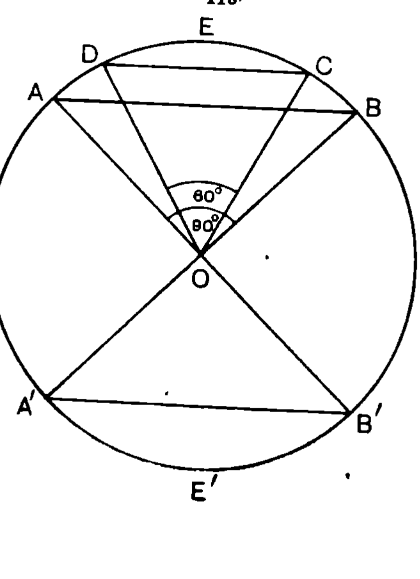
$\therefore$ OC or diámeter $= \frac{AC\times AC}{CD}$

$= \frac{7\frac{1}{2}\times7\frac{1}{2}}{\sqrt{(7\frac{1}{2})^2-(4\frac{1}{2})^2}}$

$$=\frac{15}{2}\times\frac{\overset{5}{\cancel{15}}}{2}\times\frac{}{\underset{2}{\cancel{6}}}=\frac{75}{8}=9\tfrac{3}{8}\text{ in.}$$

$= 9{\cdot}375$ in.

Ex. 6. The radius of a circle is 9 ft. 5 in. Find the area of a zone between two parallel chords drawn on the same side of the centre and subtending angles of 90° and 60° ($\pi = \frac{355}{113}$)



Area of the zone ABCD =

Area of segment ABE − area of segment DCE

Area of segment ABE =

$$\left(\frac{90}{360}\times\frac{\cancel{113}\times113\times355}{\cancel{113}}\right)-\frac{113\times113}{2}$$

$= 10028\frac{3}{4} - 6384\frac{1}{2}$

$= 3644\frac{1}{4}$

Again, area of segment DCE

$$=\left(\frac{60}{360}\times\frac{113\times113\times355}{113}\right)-\frac{\sqrt{3}}{4}\times113^2$$
$$=6685\tfrac{5}{6}-5528{\cdot}977$$
$$=1156{\cdot}856$$

$\therefore$ Area of the zone $= 3644\tfrac{1}{4}-1156{\cdot}856$
$= 2487{\cdot}394$ sq. in.
$= 17$ sq. ft. $39{\cdot}394$ sq. in.

Ex. 7. Find the area of the zone when the parallel chords in the above example are drawn on the opposite sides of the centre.

Area of the zone = Area of the circle$-$(Area of segments $DEC+A^1E^1B^1$)
(segment AEB = segment $A^1E^1B^1$)

Area of the zone $=\dfrac{113\times113\times355}{113}-(3644\tfrac{1}{4}+1156{\cdot}856)$
$= 40115-4801{\cdot}106$
$= 35313{\cdot}894$ sq. in.
$= 245$ sq. ft. $33{\cdot}894$ sq. in.

Ex. 8. The sides of a triangle measure 70, 88, 150 ft. respectively. Find the radius of the inscribed circle.

In Article 28 (footnote*) it has been proved that the area of a triangle is equal to sr when s is the half perimeter and r is the radius of the inscribed circle.

$$\therefore\ r=\frac{\text{Area of triangle}}{s}$$

Here, $r=\dfrac{\sqrt{154\times84\times66\times4}}{154}$

$$=\frac{\sqrt{14\times11\times14\times6\times6\times11\times2\times2}}{154}$$
$$=\frac{14\times11\times6\times2}{154}$$
$$=12\text{ ft.}$$

Ex. 9. The sides of a triangle measure 42, 144, 150 ft. respectively. Find the radius of the circumscribed circle.

$$\text{Radius}=\frac{\text{product of sides}}{4\times\text{area of triangle}}$$

Here,

$$\text{Radius} = \frac{42 \times 144 \times 150}{4\sqrt{168 \times 126 \times 24 \times 18}}$$

$$= \frac{42 \times 144 \times 150}{4\sqrt{21 \times 8 \times 21 \times 6 \times 8 \times 3 \times 6 \times 3}}$$

$$= \frac{\overset{2}{\cancel{42}} \times \overset{18}{\cancel{144}} \times \overset{75}{\cancel{150}}}{\underset{2}{\cancel{4}} \times \cancel{21} \times \cancel{8} \times \cancel{6} \times \cancel{3}} = 75 \text{ ft.}$$

Exercise 7

(Take $\pi = \frac{22}{7}$, unless otherwise stated.)

1. Find the circumference of the circles having the following diameters:

   (i) 42 in.
   (ii) 16 yd. 1 ft.
   (iii) 2 chains 52 links.
   (iv) 1 rasi 8 kathas.

2. Find the diameters of the circles having the following circumferences:

   (i) 5 ft. 6 in.
   (ii) 29 yd. 1 ft.
   (iii) 34 yd. 8 in.
   (iv) 232 chains 21 links.

3. The diameter of one wheel of a bicycle is 2 in. greater than the diameter of the other wheel, and the first wheel is found to make 48 revolutions less than the other wheel in covering a distance of one mile. Find the diameter of each wheel.

4. Find the cost of fencing a circular grass plot of radius 49 ft., at the rate of 8 annas a yard.

5. A man runs a quarter of a mile in a circle. Find the radius of the circle.

6. At what rate must a bicycle travel in order that its driving-wheel of 28 in. diameter may make 144 revolutions per minute?

7. Find the areas of the circles having the following radii:

   (i) 17 yd. 1 ft. 6 in.
   (ii) 2 chains 10 links
   (iii) 3 chains 8 links
   (iv) 4 yd. 2 ft. 7 in.

8. Find the radii of the circles having the following areas :

(i) 154 sq. ft.

(ii) 10 sq. ft. 100 sq. in.

(iii) 6·16 sq. chains.

9. A circular grass plot of 42 ft. radius has a path round it of a uniform width of 4 ft. Find the cost of paving the path at the rate of 6 annas per sq. yd.

10. From a square sheet of metal a circular portion is cut. If the side of the square is 10 ft., and the radius of the circle 3 ft. 6 in., find the value of the remainder at the rate of 1*s*. 2*d*. per sq. ft.

11. The radius of the outer circle of a ring is 21 ft., and the area of the ring is 770 sq. ft. Find the radius of the inner circle.

12. The height of an arc is 9 in. and the chord of the arc is 2 ft. 3 in. Find the diameter of the circle.

13. The chord of an arc is 4 chains 40 links, and the chord of half the arc is 2 chains 21 links. Find the height of the arc.

14. The diameter of a circle is 12 chains 50 links, and the height of an arc is 1 chain 25 links. Find the length of the chord.

15. The radius of a circle is 2 ft. 8 in. Find the length of an arc which subtends an angle of 30° at the centre.

16. The radius of a circle is 14 ft. Find the length of the arc which subtends an angle of 45° at the centre.

17. The radius of a circle is 2 ft. 11 in. Find the angle subtended at the centre by an arc of 2 ft. 9 in.

18. The length of an arc is 2 chains 31 links, and it subtends an angle 6° 45′ at the centre. Find the radius of the circle.

19. The chord of an arc is 6 chains 24 links, and the chord of half the arc is 4 chains 32 links. Find the length of the arc. (Use Huygens' rule.)

20. The chord of an arc is 2 ft. 8 in. and the height of the arc is 1 ft. Find the length of the arc. (Use Huygens' rule.)

21. In a circle of radius 85 ft. there are two parallel chords whose lengths are 72 ft. and 102 ft. respectively. Find their distance apart.

22. A ship steams due south at the rate of 14 miles an hour. Through how many degrees of latitude will she have passed in two days, given that the earth's mean diameter is 7912 miles ?

23. Find the area of a sector whose angle measures 60° and the radius of the circle is 1 ft. 4 in.

24. In a circle of radius 13 chains 50 links, find the area of a sector whose angle measures 13° 7′ 30″.

25. The area of a sector is 36 sq. in., the angle of the sector is 70°. Find the radius of the sector.

26. Find the area of a sector whose radius measures 15 in. and arc 28 in.

27. The area of a sector is 160 sq. ft., the radius is 16 ft. Find the angle of the sector.

28. Find the length of an arc of a sector whose area measures 33 sq. ft., and radius 6 ft.

29. The radius of a circle is 10 in. and the angle measures 90°. Find the area of the segment.

30. Find the area of a segment of a circle of radius 2 ft. 6 in., if the chord of the segment subtends an angle of 60° at the centre of the circle.

31. The radius of a circle is 8 chains 40 links. Find the area of a segment whose chord is equal to the radius.

32. Find the area of the greatest sector that can be cut from an equilateral triangle of 10 in. side., the centre of the sector being at a vertex of the triangle.

33. Find the area of a segment of a circle of radius 10 in., the chord subtending at the centre an angle of 120°.

34. In a circle of radius 10 ft., find the area of the zone between two parallel chords on the same side of the centre and subtending angles of 90° and 60° at the centre respectively.

35. In a circle of radius 16 ft., find the area of the zone between two parallel chords drawn on opposite sides of the centre and subtending angles of 90° and 120° respectively.

36. Find the radii of the inscribed and circumscribed circles to a triangle whose sides measure 48, 60, and 36 ft. respectively.

37. Find the radii of the inscribed and circumscribed circles to a triangle whose sides measure 72, 58, and 50 yd., respectively.

38. Find the areas of the circles inscribed in and circumscribed about an equilateral triangle whose side measures 2 ft. 6 in.

Examination Questions

39. A circular grass plot 40 ft. in radius is surrounded by a ing of gravel. Find the width of the gravel so that the area of the rass and gravel may be equal.

40. Find the area of the ring, the outer and inner radii of which re 3 yd. and 5 ft. respectively, by a method other than that of ubtracting the area of one circle from that of the other.

41. Find in yards correct to three places of decimals, the radius f a circle which encloses an acre.

42. Find the expense of paving a circular court 80 ft. in diameter, t 3*s*. 4*d*. per sq. ft., leaving in the centre a space for a fountain n the shape of a hexagon, each side of which is 1 yd. ($\pi = 3{\cdot}1416$.)

43. The area of a rectangular field is three-fifths of an acre, and ts length is double its breadth. Determine the lengths of its sides pproximately.

If a pony is tethered to the middle point of one of the longer ides, find the length of the tether in yards, correct to two places f decimals, in order that he may graze over half the field. ($\pi = 3{\cdot}1416$.)

44. The area of a circle is 385 acres. Find its circumference.

45. The radii of two circles are 6 and 8 ft., respectively. Find the radius of a circle whose area is equal to the sum of the areas of the two circles.

46. Assuming that the circumference of a circle is 3·1416 times the diameter, determine, as far as four decimal places, the radius of the circle of which the circumference is 2 fur. and 60 yd.

47. The radius of the outer circle of a ring is 342 ft., and the radius of the inner circle is half of that. Find the area of the ring.

48. The area of a circle is 50 sq. yd. Find the radius.

49. A road runs round a circular grass plot; the outer circumference is 500 yd., and the inner circumference is 300 yd. Find the area of the road.

50. The circumference of a circle is 100 ft. Find the length of the side of the inscribed square. The ratio of the circumference to the diameter is 3·14159 : 1. (Answer to be correct to two decimal places.)

51. A two-wheeled carriage, whose axle-tree is 4 ft. long is driven round a circle, the outer wheel makes one and a half revolutions for every single revolution of the inner one; the wheels are each 3 ft. high. What is the circumference of the circle described by the outer wheel?

52. A man, by walking diametrically across a circular grass plot, finds that it has taken him 45 seconds less than if he had kept to the path round the outside; if he walks 80 yd. a minute, what is the diameter of the grass plot?

53. The radius of a circle is $4\sqrt{2}$ ft. Find the difference in area between the square inscribed in the circle and the circle inscribed in the square.

54. A regular hexagon is inscribed in a circle of radius 1 ft. Compare the areas of the hexagon and the circle.

55. The difference between the circumference and diameter of a circle is 60 ft. Find the radius.

56. A circular grass plot is surrounded by a ring of gravel b feet wide: if the radius of the circle, including the ring, be a feet, find the relation between a and b so that the areas of grass and gravel may be equal.

57. A garden in the shape of a trapezoid, whose parallel sides are 1000 and 900 yd., and the length 800 yd., has an elliptical pond in its centre, whose diameters are 300 and 400 yd. respectively. How many square poles are available for cultivation? (Note—Area of ellipse $= \pi.ab$, where a and b are the semi-diameters.) ($\pi = 3{\cdot}1416$.)

58. A circular grass plot, whose diameter is 40 yd., contains a gravel walk 1 yd. wide, running round it 1 yd. from the edge. Find what it will cost to turf the grass plot at 4*d*. per sq. yd.

59. What will be the expense of paving a circular court of 30 ft. diameter, at 2*s*. 3*d*. per sq. ft., leaving in the centre a hexagonal space of $3\frac{1}{2}$ ft. side?

60. Two men, A and B, purchase a grindstone 1 yd. in diameter for Rs. 15, of which the first pays Rs. 8, and the other Rs. 7; now supposing the axle hole to be 1 ft. in diameter, how many inches of the radius may A grind down before sending it to B?

61. Supposing the earth were spherical, and that its circumference measured 25000 miles, the distance from Saharunpore to Agra being about 200 miles, find how high vertically a man must ascend at one of these places in order to see the other. ($\pi = 3{\cdot}1416$.)

62. A circular grass plot, whose diameter is 70 yd., contains a gravel walk 5 yd. wide round it, 15 yd. from the edge. Find what it will cost to turf the grass plot at Rs. 2 a sq. yd.

63. A road runs round a circular shrubbery; the outer circumference is 500 ft., and the inner circumference is 420 ft. Find the area of the road.

64. What is the area of the largest circle that can be inscribed in a square whose area is 5,499,025 sq. ft.? Give also the length of its circumference.

65. In cutting four equal circles, the largest possible, out of a piece of cardboard 10 in. square, how many sq. in. must necessarily be wasted. ($\pi = 3{\cdot}1416$.)

66. If a circle has the same perimeter as a triangle, the circle has the greater area. Verify this statement in the case where the sides of a triangle are 9, 10 and 17 ft.

67. The radius of the inner boundary of a ring is 14 in., the area of the ring is 100 sq. in. Find the radius of the outer boundary.

68. A road runs round a circular plot of ground; the outer circumference of the road is 44 yd. longer than the inner. Find the breadth of the road.

69. A horse is tethered by a chain fastened to a ring which slides on a rod bent into the form of a triangle. Find the area outside the triangle over which he can graze, the sides of the triangle being 30, 40, and 50 ft. respectively, and the length of the chain 15 yd.

70. Find in feet, to three places of decimals, the radius of a circle the area of which is equal to the area of a regular hexagon, the side of which is 2 ft. ($\pi = 3{\cdot}1416$.)

71. The difference between the areas of two squares inscribed and circumscribed about a circle is 338 sq. ft. Find the radius of the circle.

72. The chord of an arc is 5 ft., and the diameter of the circle is 7 ft. Find the height of the arc in inches to four decimal places.

73. The chord of an arc is 100 ft., and the angle subtended by it on the circumference is 150°. Find the radius of the circle, the height of the arc and the chord of half the arc.

74. The tops of two vertical rods on the earth's surface, each of which is 10 ft. high, cease to be visible from each other when they are 8 miles apart. What is the earth's radius?

75. O is the middle point of a straight line AB, 10 in. long, and from O as centre a circle is described with radius 7 in.; P is a point on the circumference, such that PA = 5 in. Find PB.

76. The area of an equilateral triangle, whose base falls on the diameter, and its vertex in the middle of the arc of a semicircle, is equal to 100. What is the diameter of the semicircle?

77. Given the chord 20 ft. and height 4 ft. of an arc of a circle, find the diameter and length of arc.

78. AB and AC are the chords of a circle at right angles to one another; their lengths are 30 ft. and 40 ft. respectively. Find the height of the arc AC, and the diameter of the circle.

79. The radius of a circle is 7 ft. Find the perpendicular from the centre on a chord 8 ft. long.

80. The chord of an arc is 49 ft. and the chord of half the arc is 25 ft. Find the diameter of the circle.

81. Two parallel chords in a circle are 6 in. and 8 in. long, and 1 in. apart. Find the diameter.

82. The diameter of a circle is 12 ft. Find the side of the square inscribed in it.

83. Find the area of a square inscribed in a quadrant whose radius is $\sqrt{3}$, two sides of the square being coincident with the radii.

84. The span of a bridge, the form of which is an arc of a circle, being 96 ft., and the height 12 ft., find the radius.

85. The width of a circular walk is 4 ft., and the length of the line which is a chord of the outer circumference and a tangent to the inner circumference is 20 ft. Find the area of the walk. ($\pi = 3{\cdot}14159$.)

86. A lighthouse is to be constructed at a distance of 42 miles from a port; how high should it be in order that the light may be just visible from the port, taking the average height of a man as 6 ft.?

87. Three circles, each of radius 1 ft. touch each other. Find the area of the curvilinear figure included between them. ($\pi = 3{\cdot}14159$.)

88. A circular disc of cardboard 1 ft. in diameter is divided into six equal sectors, by pencil lines through the centre. In each sector there is described a circle touching the two bounding radii of the sector, and also the arc joining their ends at its middle

point. If the circles are cut out from the six sectors, find the area of cardboard remaining.

89. A chord of a circle subtends an angle of 60° at the centre: if the length of the chord be 100, find the areas of the two segments into which the chord divides the circle.

90. A field in the form of an equilateral triangle contains half an acre. What must be the length of a tether fixed at one of its angles and to a horse's nose to enable him to graze exactly half of it?

91. Find the area of a sector when the radius is 50 ft., and the length of arc 16 ft.

92. Find the area of the space which is common to four equal circles which intersect each other, their centres being at the angular points of a square, and their radii equal to a side of the square.

93. The radius of a circle is 75; a zone of that circle has one of its parallel chords coinciding with the diameter, and the other equal to the radius: what is the area of the zone? ($\pi = 3{\cdot}14159$.)

94. The circumference of a circle is 11 ft. Find the length of the radius, and the area of the segment cut off by a chord equal to the radius.

95. The radius of a circle is 25 ft.; two parallel chords are drawn, each equal to the radius. Find the area of the zone between the chords. ($\pi = 3{\cdot}14159$.)

96. Find the diameter of the circle round a triangle whose sides are 123, 122, and 49.

97. Given a circle of radius 1 ft., find to three places of decimals the side of an equilateral triangle inscribed in it.

98. Find the diameter of the circle circumscribing a triangle, the sides of which are 68, 285, and 293 ft. respectively.

99. Two sides of a triangle containing an obtuse angle are equal to 10 and 14 in. respectively, and the perpendicular on the third side from the vertex is equal to 7 in. Find the diameter of the circumscribing circle.

100. The three sides of a triangle inscribed in a circle are 120, 160, and 180 ft. respectively. Find the difference between the area of the circle and the area of the triangle.

101. Find the area in square chains of the circle inscribed in a triangle of which the sides are 372, 350, and 320 yd. respectively.

CHAPTER XI

AREA OF SIMILAR FIGURES

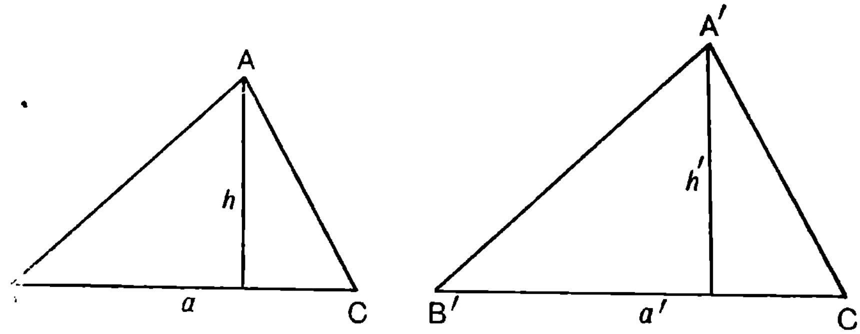
48. Similar figures may be described as figures of the same *shape*, but not necessarily of the same *size*.

Thus circles of all sizes are similar figures.

The ratio of the areas of two similar plane figures is equal to the ratio of the squares of two corresponding lengths in the two figures.

49. As a particular case we can show that the proposition stated in the preceding paragraph holds for similar triangles:

Let ABC and $A^1B^1C^1$ be two similar triangles. Denoting the bases by a and a^1, and the heights by h and h^1, we have $\frac{a}{a^1} = \frac{h}{h^1}$:



Also $\frac{\text{Area of ABC}}{\text{Area of } A^1B^1C^1} = \frac{\frac{1}{2}ah}{\frac{1}{2}a^1h^1} = \frac{ah}{a^1h^1} = \frac{a^2}{(a^1)^2}$ or $\frac{h^2}{(h^1)^2}$

Instead of bases or heights we may substitute any other corresponding lengths such as the radii of the circumscribed circles.

50. The irregularly curved boundary of a country and its representation on a map are similar figures.

If in a map a distance of one mile on the ground is represented by a length of 8 in. then the ratio should be

$$\frac{\text{Area on the ground}}{\text{Area of the map}} = \frac{\text{one mile}^2}{8 \text{ inches}^2}$$

Illustrated Examples

Ex. 1. In two similar triangles the bases are 11 ft. and 17 ft., respectively. If the area of the first is 66 sq. ft. what is the area of the second ?

Area of the second : 66 sq. ft. $= 17^2 : 11^2$

$$\therefore \text{ Area of second} = \frac{66\times17\times17}{11\times11} = \frac{1734}{11}$$
$$= 157\tfrac{7}{11} \text{ sq. ft.}$$

Ex. 2. The ground plan of a house drawn to a scale of one in. to 8 ft. is represented by a rectangle 8 in. by 6 in.; what area is covered by the basement of the house ?

Area of the basement : 8×6 sq. in. $= (8 \text{ ft.})^2 : (1 \text{ in.})^2$

$$\text{Area} = \frac{8\times6\times8\times8\times144}{1} = 3072\times144 \text{ sq. in.}$$
$$= 3072 \text{ sq. ft.}$$

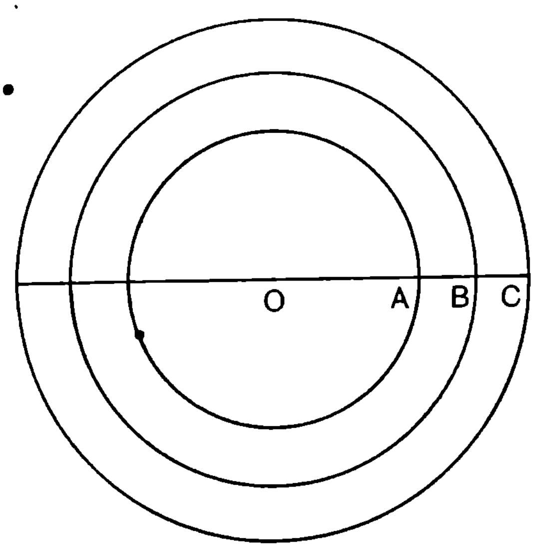
Ex. 3. In a plan every square inch of surface represents an area of 10 acres. Find the scale of the plan.

In this case it is required to find out what length of 1 in. on the plan represents on the ground.

Then $(1 \text{ in.})^2 : (\text{length on the ground in inch})^2 = 1 \text{ sq. in.} : 10 \text{ acres}$

$$\therefore \text{ (length on the ground in inch)} = \sqrt{10\times4840\times9\times144}$$
$$= 10\times22\times3\times12$$
$$= 7920 \text{ inches.}$$

$\therefore$ one inch on plan represents 7920 inches on ground, i.e. the scale of the map is 1 to 7920 or 8 inches to 1 mile.



Ex. 4. A circle of 30 yd. radius is divided into three equal parts by drawing two concentric circles. Find the radii of these circles.

Let OC be the radius of the given circle and let OA and OB be the radii of the concentric circles. Let OA measure r and OB measure R

$90^2 : r^2 = 3 : 1$

$$\therefore\ r = \sqrt{\frac{90 \times 90}{3}}$$

$$= \frac{90}{\sqrt{3}} = 30\sqrt{3}. \doteq 51{\cdot}96 \text{ ft.}$$

Hence $R = 30\sqrt{3} \times \sqrt{2} = 30\sqrt{6} = 73{\cdot}48$ ft.

The radii are 73·48 ft. and 51·96 ft.

Ex. 5. The sides of a triangle are in the proportion of 26, 28, and 30. Its area is 3024 sq. ft. Find the three sides.

The area of a triangle with 26, 28 and 30 as sides =

$\sqrt{42 \times 16 \times 14 \times 12} = 14 \times 4 \times 6 = 336$

First side$^2 : 26^2 = 3024 : 336$

$$\therefore \text{ First side} = \sqrt{\frac{26 \times 26 \times 3024}{336}} = 78 \text{ ft.}$$

Hence 2nd and 3rd sides are 84 and 90 ft. respectively.

Exercise 8

1. If the area of one triangle is nine times the area of a similar triangle, and if the base of the first triangle measures 9 in., find the base of the second triangle.

2. A plan of a field is drawn to a scale of 1 in. to 8 ft.; if the field contains 640 sq. yd., how many square inches will the plan occupy ?

3. The sides of a rectangle are in the proportion, 12 and 13, and the area is 624 sq. in. Find the sides.

4. In a settlement map a square inch represents 2·5 acres. Find the scale of the map.

5. A circle of 100 ft. radius is divided into two parts of equal area by a concentric circle. Find the radius of the latter.

Examination Questions

6. A circle of 120 ft. radius is divided into three parts by two concentric circles. Find the radii of these circles so that the three parts may be of equal area.

7. The ratio of the similar sides of two similar triangles is 13 : 17. Find the ratio of their areas.

8. What will be the size of the map of a district which contains 676 sq. miles, on a scale of 4 miles to the inch ?

9. The sides of a triangle are 20, 13, and 21 ft.; a straight line is drawn from the middle point of the side of 20 ft. across the triangle parallel to the longest side. Find the area of the two parts into which the triangle is divided.

10. Of two concentric circles the area of the smaller is half that of the larger. Find the radius of the larger circle, that of the smaller being 4 ft.

11. Find the scale to which a plan is drawn, 1 sq. in. representing 10 acres.

12. What is the scale to which a plan is drawn if 1 sq. in. represents 1 acre.

13. The sides of a triangle are 39, 52, and 65 ft. respectively. Find the sides of a similar triangle of nine times its area.

14. One side of a field measuring 28 acres and 9 cents is 17 chains. What is the area of a similar field whose corresponding side measures 27 chains ?

15. If from a right angled triangle whose base is 12 and perpendicular height 16 ft. a line be drawn parallel to the perpendicular, cutting off a triangle whose area is 24 sq. ft., what are the sides of this triangle ?

16. The radius of a circle is 20 in.; it is required to draw three concentric circles in such a manner that the whole area may be divided into four equal parts. Find the radii.

17. Out of a circular disc of metal 35 equal holes are punched; the weight of metal thus punched out is to the weight of the perforated disc as 45 : 67. Compare the diameters of the disc and of the holes, given that the area of a circle varies as the square of the diameter.

18. The radius of a circle is 18 in. Find the radius of another circle of one-fifth the area.

19. The area of a rectangular plot, whose length is equal to twice its breadth, is 3528 yd. If a gravel path 4 ft. wide goes along one of the diagonals of the plot, find the area of the path.

20. Find the height of a triangle which shall be similar to, and contain five times as much as, another 50 ft. in altitude.

21. Find the dimensions in feet of a trapezoid which shall contain 100 sq. yd., similar to one whose parallel sides are 8 and 10, and perpendicular distance between them, 4.

22. On a map drawn to the scale of 1/10000 the sides of a rectangular field are 0·65 and 0·72 in. Find the area of the field in acres, and the length of the diagonal in yards.

23. One side of a triangle is 20 ft., divide the triangle into five equal parts by straight lines parallel to one of the other sides, and find the distances from the vertex of the points of division of the given side.

24. A circle whose area is 314·16 sq. in. is to be divided into four equal portions by concentric circles. Find their diameters. ($\pi = 3{\cdot}1416$.)

25. The sides of a triangle are 532, 427, and 389 ft. Find the length of a line, parallel to the longest side, that will divide the triangle into two equal parts.

26. A drawing is copied to a scale one half as large again as the original scale; in what ratio is its surface augmented ?

27. The sides of a triangular field are 3501, 3604, 3605 ft. respectively. Find the length of a line drawn parallel to the longest side which will divide the field into two equal parts.

28. A circular hole is to be cut in a circular plate so that the weight may be reduced one third. Find the diameter of the hole.

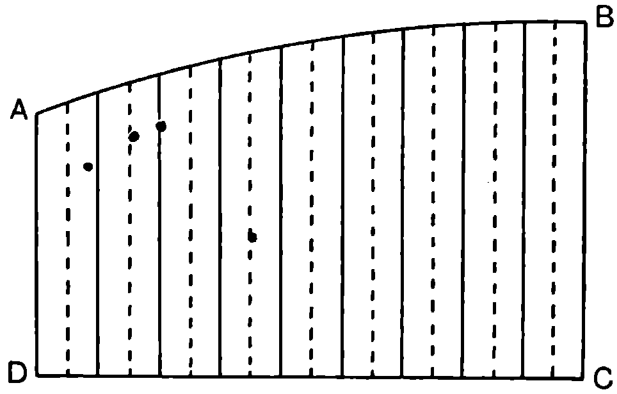
29. A board is 12 in. wide at one end, and 9 in. at the other end, and its length is 8 ft. How far from the broad end must it be sawn across so as to divide the plank into two equal portions ?

30. Find the dimensions of a triangle similar to one whose dimensions are 50, 60, and 80 ft. but which shall contain three times the quantity.

Chapter XI

APPROXIMATE RULES FOR AREAS

51. Mid-Ordinate Rule

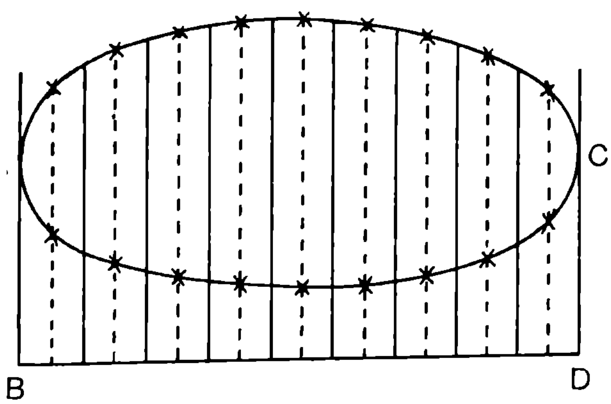


Let AB be any curve, DC a straight line and DA and CB two ordinates drawn at right angles to DC. The area of ADCB is required. Divide DC into a convenient number of *n* equal parts and draw ordinates to the curve at the points of division, thus dividing the area into *n* strips of equal breadth. Draw the middle ordinates of each strip as shown in dotted lines. If we denote the lengths of these mid-ordinates by h^1, h^2, h^3 . . . h^n then the area of ADCB will be $= \frac{DC}{n} \times (h^1 + h^2 + h^3 + \ldots h^n)$ or

Area $=$ DC $\times$ mean height.

A close examination of this rule with the diagram will show that it is based on the principle of *give and take.* If each section of the curve AB be straight lines the above formula would be exact. If the strips are narrow the sections will be very nearly straight and the result gives a close approximation of area.

52. The mid-ordinate rules may be applied for finding the area of a closed curve in the following manner:



Draw parallel tangents AB and CD and draw BD at right angles to the tangents. Divide BD in *n* parts, draw mid-ordinates as shown in dotted lines. Measure the mid-ordinates as each is terminated by the curve as indicated by x. Now the area will be = total length of mid-ordinate $\times \frac{BD}{n}$. Instead of measuring each mid-ordinate separately, take a long strip of paper and by placing it lengthwise against the strips, mark off the lengths of the mid-ordinates in succession and so obtain the total length of the mid-ordinates by a single measurement.

53. The best known rule for calculating the approximate area of a curve was discovered by Thomas Simpson, which rule is known as the Simpson's Rule.* To obtain area by this rule of the figure given in Art. 51, divide DC into an even number (say, 8) equal parts and draw the ordinates at points of divisions. There will thus be 9 ordinates in all and the area is $\frac{DC}{3\times 8}$ [1st ordinate+last ordinate+4(sum of even ordinates)+2(sum of odd ordinates).]

Hence the rule is

"Add together the 1st and last ordinates, 4 times the sum of even ordinates, and twice the sum of odd ordinates; multiply the result by one-third the common distance."

Simpson's Rule may be applied to a closed curve by drawing parallel tangents and then drawing intermediate ordinates parallel to the tangents in the same way as the Mid-ordinate Rule. In this case, however, it should be remembered that the first and last ordinates are the tangents and are zero.

Simpson's Rule is extensively used in ship-building for finding the areas of vertical or horizontal sections of the hull of a ship, and ultimately for finding the volume of the hull under the water-line and hence the displacement of the ship.

Examples

Ex. 1. Find the area of a curvilinear figure in which the mid-ordinates drawn 1 ft. apart are 4, 7, 8, 10, 7, 6, 3 ft.

$$\text{Area} = 1\times(4+7+8+10+7+6+3)$$
$$= 45 \text{ sq. ft.}$$

* Thomas Simpson (born August 20, 1710, d. May 14, 1761) was the son of a weaver of Leicestershire. He was self-educated and his mathematical interests were aroused by the solar eclipse which took place in 1724. In 1743 he was appointed professor at the Royal Military Academy, Woolwich, which post he held till his death.

Ex. 2. Find the area by Simpson's Rules having given the llowing:

Ordinates O, 1·5, 2·25, 3, 2·75, 1·25, 0 ft., common distance 5 ft.

$$\text{Area} = \frac{1\cdot5}{3}[4(1\cdot5+3+1\cdot25)+2(2\cdot25+2\cdot75)]$$

$$= \frac{1\cdot5}{3}\times[23+10]$$

$$= 16\cdot5 \text{ sq. ft.}$$

Ex. 3. A river is 240 ft. wide, and is bounded on one side by a rtical wall 19 ft. deep. Soundings taken 20, 40, 60, etc., away om the wall give depths in feet of 19·4, 20·2, 18, 17·5, 14·9, 11, 2, 5·1, 4·4, 2·4, 1·1, 0. Find the area of cross section of the rer.

$$\text{rea} = \frac{20}{3}[19+0+4(19\cdot4+18+14\cdot9+7\cdot2+4\cdot4+1\cdot1) +2(20\cdot2+17\cdot5+11+5\cdot1+2\cdot4)]$$

$$= \frac{20}{3}[19+260+112\cdot4]$$

$$= \frac{20}{3}\times391\cdot4 = \frac{7828}{3}$$

$$= 2609\cdot\dot{3} \text{ sq. ft.}$$

$$= 2609 \text{ sq. ft. } 48 \text{ sq. in.}$$

Exercise 9

1. Find the area of a curvilinear figure which is divided into equal strips of 1 ft. 8 in. breadth. The mid-ordinates measure ł, 3·2, 2·8, 4·2, 3·4, 3·6, 4·2, and 2·6 respectively.

2. Find the area of a field enclosed by a curve and ordinates)82 ft. and 6·557 ft., when 5 intermediate ordinates measure 24 ft., 6·245 ft., 6·325 ft., 6·403 ft., and 6·481 ft., the common erval being 18 in. (Apply Simpson's Rule.)

3. To determine the area of a given oval curve two parallel igents are drawn, and between these, nine equidistant chords rallel to the tangents. If the distance between the tangents 4·45 in. and the lengths of the chords are 1·475, 2·05, 2·275, 5, 2·625, 2·55, 2·375, 2·075, and 1·55 in. respectively, find the a of the curve.

4. One side of a field AB is 80 ft. long, two other sides, AC and BD at right angles to it, are respectively 16 and 60 ft., the fourth side is curved, and the ordinates to it from AB (at intervals of 10 ft. from A) are 18, 24, 36, 21, 28, 40, and 50 ft. What is the area of the field ?

5. Find the area of a field having the following dimensions: ordinates 0, 20, 32, 36, 32, 20, 0 ft., the common distance being 20 ft.

Examination Questions

6. Thirteen equidistant ordinates are measured to a curve at 100 ft. intervals on a chain line. Find the area between the end ordinates, the curve and the chain line. The ordinates are: 50, 60, 80, 90, 30, 50, 60, 80, 70, 90, 100, 120, 130.

7. The length of a line is 584 ft. and at equal distances along it the following offsets were taken to an irregularly curved fence, viz. 93, 84, 72, 68, 43, 54, 37, 29, and 23 ft. Find the area included between the extreme offsets, the fence, and the chain line.

8. Apply Simpson's rule to find the area of a section, the heights of which above the railway level, at intervals of 30 ft., are 2, 10, 15, 20, 30, 25, 17·5, 10, 3 ft.

9. Describe what is meant by "Simpson's Rule." Under what circumstances is it applicable ?

10. Find by Simpson's Rule the area of a curvilinear figure whose ordinates are 9, 11, 13, 12, 10, 13, 15, 17, 14, 12, 7 ft.; base = 73 ft.

Chapter XIII

CUBE ROOT

54. Certain questions in solid mensuration require the extraction of cube root.

Table :

Number :	1	2	3	4	5	6	7	8	9
Cubes :	1	8	27	64	125	216	343	512	729

Arrange three columns headed 0, 0, and the number whose cube root is required as under :

0 0 373′248

Divide the digits in the number into groups of three, beginning from the right as shown above. The cube root will consist of a number of digits as the number of groups. The cube root of the above number will be of two digits.

The first figure of the cube root is the highest number whose cube is less than the number in the left hand group. In the above number the left hand group is 373 and 7 is the highest number whose cube is less than this. The first digit in the cube root is therefore 7.

Let this digit be called d.

Now continue the operation as follows :

(i) Add d to the figure in the first column and place the same in the first column, as $(0+7 = 7)$ in the example below.

(ii) Multiply the sum by d and add the product to the figure in the second column and place the sum in the second column $(7\times7 = 49)$.

(iii) Multiply the sum in the second column by d and subtract the product from the first group of the number.

The first stage of operation is now finished.

Second stage :

(iv) Bring down the next group. Thus in the example the figure to be dealt with now is 30248.

Repeat processes (i) and (ii), e.g. :

(i) Add d to the last figure in the first column and place the sum $(7+7 = 14)$ in the first column.

(ii) Multiply this sum by d and add the product to the last figure in the second column and place the sum in the second column. Thus in the example below the figure now in the second column is $147(14\times7+49)$. Place two zeros against this sum. Add d again to the last figure in the first column and place one zero against it. Thus in the example the figure now in the first column is 210 and that in the second column is 14700.

To find an approximation to the second digit in the cube root divide the number in the third column by the number in the second column. This will never be too small, but may be too great. Thus the second figure in the example is $(30248\div14700=)$ 2.

Now, repeat the processes (i), (ii), and (iii).

[Remember that d is now 2.]

In the example $(210+2 =)$ 212 in the first column.

In the second column $(212\times2+14700 =)$ 15124.

In the third column $(15124\times2 =)$ 30248.

	0		0		373 248(72
$(0+7 =)$	7	$(7\times7 =)$	49	(49×7)	343
$(7+7 =)$	14	$(14\times7+49 =)$	14700		30248
$(7+14 =)$	210				
$(210+2 =)$	212	$(212\times2+14700=)$	15124	$(15124\times2=)$	30248*

55. The students should carefully study the following examples :

Ex. 1. Extract the cube root of 7245075375.

* If this figure is larger than the figure above, the trial digit is too large, and a lower one must be tried.

	0		0		7'245'075'375(1935
(0+1)	1	(1×1 =)	1	(1×1 =)	1
(1+1)	2	(2×1+1 =)	300		
(1+2)	30				6245
(9+30)	39	(39×9+300 =)	651	(651×9 =)	5859
(9+39)	48	(48×9+651 =)	108300		
(9+48)	570				386075
(3+570)	573	(573×3+108300 =)	110019	(110019×3 =)	330057
(3+573)	576	(576×3+110019 =)	11174700		56018375
(3+576)	5790				
(5+5790)	5795	(5795×5+11174700 =)	11203675	(11203675×5 =)	56018375

	0		0		127'795'585'653(5037
(0+5 =)	5	(5×5 =)	25	(25×5 =)	125
(5+5 =)	10	(10×5+25 =)	7500		
(10+5)	150		750000		2795585
	1500				
(1500+3 =)	1503	(1503×3+750000 =)	754509	(754509×3 =)	2263527
(1503+3 =)	1506	(1506×3+754509)	75902700		532058653
(15090+7 =)	15097	(15097×7+75902700 =)	76008379	(76008379×7 =)	532058653

In this example, to find the second figure, 6245 ÷ 300 suggests 20. But this is absurd. The figure cannot be more than 9. We therefore try the digit 9 as being the highest possible. It might have been found too high. A little practice will settle this question. This difficulty will however rarely occur after the second figure of the root has been found.

Ex. 2. Find the cube root of 127795585653.

One difficulty arises here for finding the second figure of the root; the trial divisor is 7500 and the dividend is 2795. The divisor will not divide the dividend, the figure is therefore 0. We write this figure and then we put one more 0 at the end of the figure in the first column and two 0's at that of the second column and bring down the next group of three figures in the third column. The trial divisor is now 750000 and the dividend 2795585 and the quotient is clearly 3.

56. The cube root of decimals is determined by similar principle, the number being divided into groups of three digits right and left starting from the decimal point. Similarly cube roots of numbers which are not perfect cubes may be calculated to any desired number of figures.

Ex. 3. Find the cube root of 10793·861

0	0	10'793·'861(22·1
2	4	8
4		
60	1200	2793
62	1324	2648
64		
660	145200	145861
661	145861	145861

Ex. 4. Find the cube root of ·002202073901

0	0	·002'202'073'901(·1301
1	1	1
2		
30	300	1202
33	399	1197
36		
3900	5070000	5073901
3901	5073901	5073901

Ex. 5. Find the cube root of ·008869743

0	0	
2	4	·008'869'743(·207
4		8
600	120000	869743
607	124249	869743

Exercise 10

Find the cube root of:

1. 32768 ; 74088 ; 226981
2. 531441 ; 857375 ; 110592
3. 726572699 ; 929714176 ; 967361669
4. 1018108216 ; 8060150125 ; 64144108027
5. 320575259584 ; 963259373376
6. ·000343 ; ·166375 ; ·287496
7. 2571·353 ; 440711·081
8. ·002744588042001
9. 4921·675101
10. 2·197507039001

Find the cube roots of the following to 4 significant figures:

11. ·8 ; ·08
12. 2·7 ; ·27
13. 4 ; 5 ; 6
14. 61 ; 81 ; 1342
15. $\cdot\dot{8}$; $\cdot 9\dot{1}$; $\cdot\dot{9}2307\dot{6}$

Chapter XIV

SOLIDS

57. A solid figure bounded by six equal squares is called a cube. If the sides of the squares were 1 in. each the solid would be one cubic inch. The bounding squares are called the faces and the sides of the squares, edges of the solid.

The volume or solid content of a body means the number of cubic units which it contains, whether those units be cubic inches, cubic feet, cubic millimetres, or, other measures.

58. The units of volume most commonly used in mensuration are given below:

1728 cubic inches (cubic in.) = 1 cubic foot (cubic ft.)

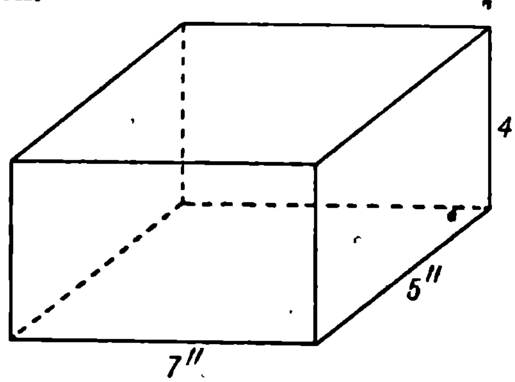
27 cubic ft. = 1 cubic yard.

One cubic foot of pure water weighs 997·137oz. (Av.), but in practice it is usual to take 1000 oz. or 62·5 lbs. as the weight of one cubic foot of pure water.

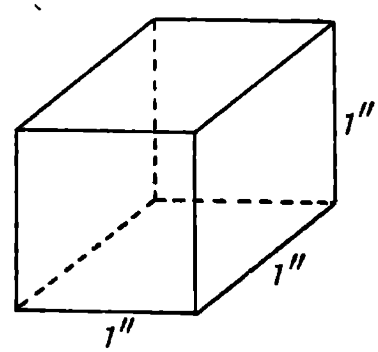
The imperial gallon contains 277·274 cubic in. But in practice 277¼ cubic in. is taken as the capacity of one gallon.

59. If each of the six bounding faces be a rectangle of any shape the figure is called a cuboid, a rectangular solid, or, a rectangular parallelopiped.

Let the solid in Fig. I measure 7 in. long, 5 in. wide, and 4 in. thick.



(Fig. i)
CUBOID



(Fig. ii)
CUBE

Now cut the solid into 4 layers each 1 in. thick.

Cut each layer lengthwise into 5 equal strips. Finally cut each strip crosswise into 7 equal pieces.

Obviously each of these pieces is a 1 in. cube, and each strip contains 7 cubic in.

Each layer contains $7 \times 5 = 35$ cubic in.

and the solid contains $7 \times 5 \times 4 = 140$ cubic in.

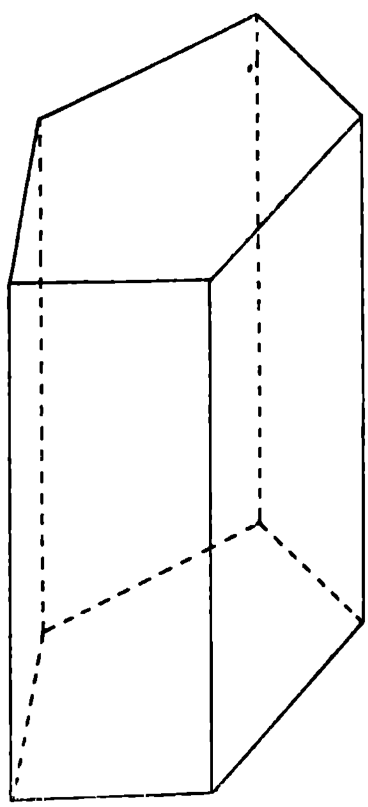
Hence the rule,

volume of a cuboid = length × breadth × thickness, or, height, or, depth.

When the three dimensions are the same as in a cube the volume = (length or edge)^3.

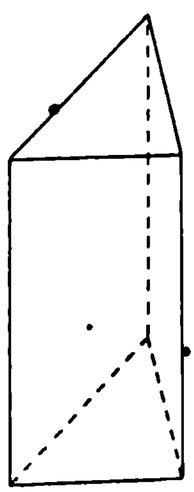
60. A prism is a solid bounded by plane faces, two of which, called the ends, are similar, equal, and parallel figures, and the others are parallelograms.

The figures below show different prisms. It will be seen that the figure II which shows a rectangular prism is the same as a cuboid or a rectangular parallelopiped.



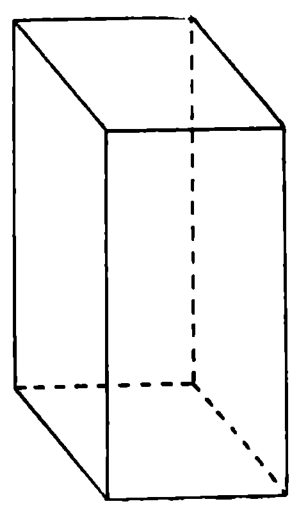
(Fig. i)

TRIANGULAR PRISM



(Fig. ii)

RECTANGULAR PRISM



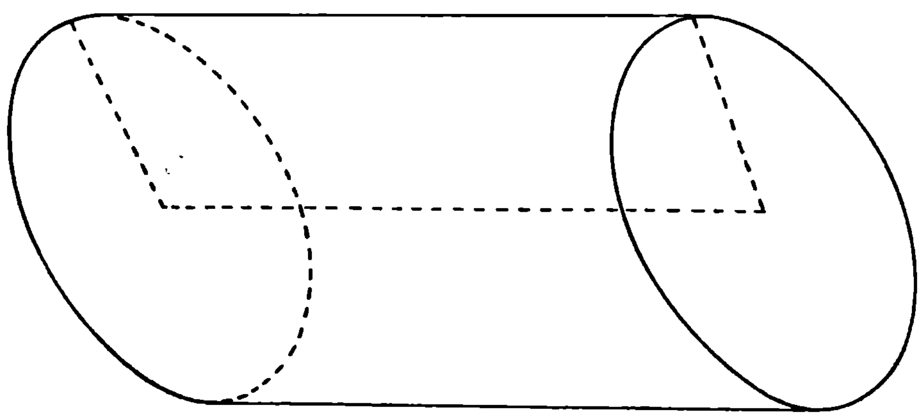
(Fig. iii)

PENTAGONAL PRISM

The idea of a prism is simply obtained by the following practical way of making one. Suppose a sheet of steel contains a hole in the form of a polygon, and suppose that this hole is provided with a cutting blade along each edge. Let a wooden rod, thick enough to fill the hole, be driven through the sheet, remaining perpendicular to it. The rod will now be converted to a prism. If the hole be a triangle, the rod will turn into a triangular prism; if the hole be a rectangle, the prism formed will be a rectangular prism, or, a cuboid.

61. Were the hole circular, the rod would become a cylinder. A cylinder may be defined as the solid formed by generating a rectangle on one of its sides.

The figure below shows a cylinder. The ends of a prism or a cylinder are also called bases. The perpendicular distance between the ends is called the height.



A prism or a cylinder is right, when the side faces are at right angles to the ends. If otherwise, it is oblique.

The volume of a prism or a cylinder is equal to:

Area of the base $\times$ height.

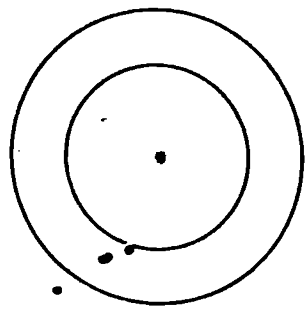
62. Circular ring: A circular ring may be roughly described as a right circular cylinder bent round in a circle until its ends meet. Since in bending the cylinder the inner portion is as much contracted as the outer portion is expanded, the volume of the ring is the same as the volume of the original cylinder. Hence the volume of a circular ring is equal to the volume of a right cylinder whose base is the same as the cross section of the ring and whose height is equal to the length of the ring.

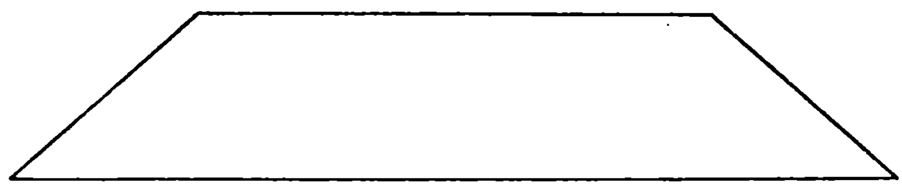
That is,

volume of a circular ring = area of cross section $\times$ length.

By the length of a ring is meant its mean circumference or circumference midway between the inner and the outer circumference.

If the ring is wrapped by a piece of paper then the size of the paper required will be as shown below:





Hence the surface area of a circular ring = mean of the inner and outer circumferences × circumference of cross section.

Examples

Ex. 1. Find the volume of a solid the edge of which measures 6 ft. 6 in.

$$\text{Volume} = 6\tfrac{1}{2} \times 6\tfrac{1}{2} \times 6\tfrac{1}{2}$$
$$= 274\tfrac{5}{8} \text{ cubic ft.}$$
$$= 274 \text{ cubic ft. } 1080 \text{ cubic in.}$$

Ex. 2. Find the volume of a rectangular prism which measures 8 ft. in length, 6 ft. in breadth, and 5 ft. in height.

$$\text{Volume} = 8 \times 6 \times 5$$
$$= 240 \text{ cubic ft.}$$

Ex. 3. Find the surface area of a cube whose edges measure 5 ft.

There are 6 surfaces, each a square, whose sides measure 5 ft.

Hence the surface area $= 6 \times 5^2 = 150$ sq. ft.

Ex. 4. What must be the depth of a reservoir whose inside length and breadth are 5 ft. 4 in., and 5 ft. 3 in., so that it may contain one ton of water? (Volume of one gallon water is 277·25 cubic in.)

$$\text{Volume of one ton water} = \frac{28\times4\times20\times277{\cdot}25}{10}\text{ cubic in.}$$

$$\therefore\ \text{Depth} = \frac{28\times4\times20\times277{\cdot}25}{10\times64\times63} = 15{\cdot}4\text{ in.}$$

$$= 1\text{ ft. } 3{\cdot}4\text{ in.}$$

Ex. 5. Find the surface and volume of a cube, in which the diagonal of each face is 1 ft. 3 in.

$$\text{The side of each face} = \frac{15}{\sqrt{2}} = \frac{15\sqrt{2}}{2}$$

$$\therefore\ \text{Area of each face} = \left(\frac{15\sqrt{2}}{2}\right)^2 = 112{\cdot}5\text{ sq. in.}$$

$$\therefore\ \text{Surface of cube} = 6\times112{\cdot}5 = 675\text{ sq. in.}$$

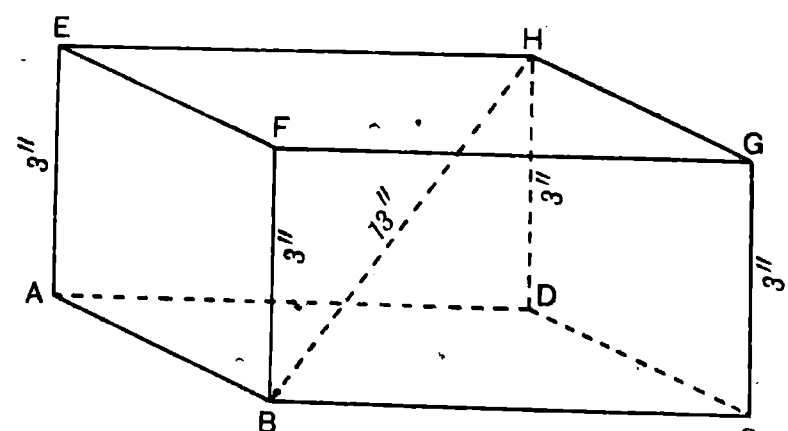
$$= 4\text{ sq. ft. } 99\text{ sq. in.}$$

And

$$\text{Volume of cube} = \left(\frac{15\sqrt{2}}{2}\right)^3 = \frac{6750\sqrt{2}}{8}$$

$$= 1193{\cdot}24\text{ cubic in.}$$

Ex. 6. Find the length and breadth of a rectangular solid, having given that the diagonal is 13 in., the height 3 in., and the area of the base is 48 sq. in.



Here given

Area of ABCD or EFGH = 48 sq. in.

BH = 13 in.

AE or BF or CG or DH = 3 in.

$$BD = \sqrt{13^2-3^2} = \sqrt{160}$$
$$= \sqrt{BC^2+AB^2}$$

$$\text{or } BD^2 = BC^2+CD^2 = 160$$

$$\text{Now } (BC+CD)^2 = BC^2+CD^2+2BC.CD$$
$$= 160+2\times 48 = 256$$
$$\therefore\ (BC+CD) = 16$$

$$\text{Again } (BC-CD)^2 = BC^2+CD^2-2BC.CD$$
$$= 160-2\times 48 = 64$$
$$\therefore\ (BC-CD) = 8$$
$$\therefore\ BC = 12, \text{ and } CD = 4.$$

Ex. 7. The base of a right prism is a right angled triangle, whose sides containing the right angle are 3 in. and 4 in. If the height of the prism is 8 in., find the volume and the whole surface.

$$\text{Volume} = \text{Area of base} \times \text{height.}$$

$$\text{i.e., volume} = \frac{4\times 3}{2}\times 8 = 48 \text{ cubic. in.}$$

$$\text{other side of the base} = \sqrt{4^2+3^2} = 5$$

$$\text{Area of surface} = 2\times\frac{4\times 3}{2}+8\times 4+8\times 3+8\times 5$$
$$= 108 \text{ sq. in.}$$

Ex. 8. The base of a prism is a right angled triangle whose hypotenuse is 17 in. If the height is 1 ft., and the sum of the rectangular faces 480 sq. in., find the other sides of the base.

Let the sides be a and b

$$\text{then } a^2+b^2 = 17^2 = 289$$

$$\text{Rect. surface} = 17\times 12+12a+12b = 480$$

$$\text{or } 12(a+b) = 480-204$$

$$\text{or } a+b = \frac{276}{12} = 23$$

$$(a+b)^2-(a^2+b^2) = 2ab$$
$$23^2-17^2 = 2ab$$
$$\therefore\ 2ab = 240$$

$$\text{Again } (a-b)^2 = (a+b)^2 - 4ab$$
$$\text{or } (a-b)^2 = 23^2-2\times 240$$
$$= 529-480$$
$$= 49$$

$$\therefore\ a-b = 7, \text{ and } a+b = 23$$
$$\therefore\ a = 15, \text{ and } b = 8.$$

Ex. 9. Find the surface and volume of a cylinder whose diameter is 2 yd. 2 ft. 2 in., and the height 1 yd. 1 ft.

Volume = Area of the base × height.

Here,

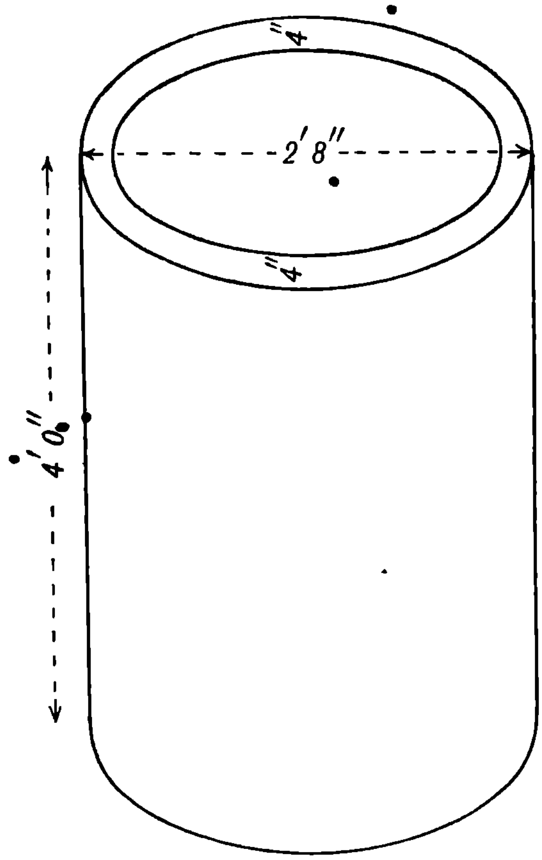
$$\text{Area of the base} = \left(\frac{2 \text{ yd. } 2 \text{ ft. } 2 \text{ in.}}{2}\right)^2 \times \frac{22}{7}$$

$$\therefore\ \text{Volume} = \frac{98}{\cancel{2}} \times \frac{\overset{\overset{7}{\cancel{49}}}{\cancel{98}}}{\cancel{2}} \times \frac{22}{\cancel{7}} \times \overset{24}{\cancel{48}}$$
$$= 362208 \text{ cubic in.}$$
$$= 7 \text{ cubic yd. } 20 \text{ cubic ft. } 1056 \text{ cubic in.}$$

$$\text{Round surface} = \text{circumference of base} \times \text{height.}$$
$$= \frac{\overset{14}{\cancel{98}}}{\underset{3}{\cancel{12}}} \times \frac{22 \times \cancel{4}}{\cancel{7}} = \frac{308}{3} = 102\tfrac{2}{3} \text{ sq. ft.}$$
$$= 11 \text{ sq. yd. } 3 \text{ sq. ft. } 96 \text{ sq. in.}$$

$$\text{Ends} = \frac{\cancel{2} \times \overset{49}{\cancel{98}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{12}} \times \cancel{2}} \times \frac{\overset{\overset{7}{\cancel{14}}}{\cancel{98}}}{12 \times \cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} = \frac{3773}{36} = 104\tfrac{29}{36} \text{ sq. ft.}$$
$$= 11 \text{ sq. yd. } 5 \text{ sq. ft. } 116 \text{ sq. in.}$$

$$\text{Whole surface} = 23 \text{ sq. yd. } 68 \text{ sq. in.}$$



Ex. 10. An iron roller is in the shape of a hollow cylinder whose length is 4 ft., external diameter 2 ft. 8 in., and thickness 4 in. Find (in pounds) its weight, supposing one cubic foot of iron to weigh 486 lb.

If the roller were a solid one the volume would be

$$= \left(\frac{2\frac{2}{3}}{2}\right)^2 \times \frac{22}{7} \times 4 = \frac{16}{9} \times \frac{22}{7} \times 4 = \frac{1408}{63} \text{ cubic ft.}$$

Internal diameter = 2 ft. 8 in. − 8 in. = 2 ft.

And the volume of the inside opening

$$= 1^2 \times \frac{22}{7} \times 4 = \frac{88}{7} \text{ cubic ft.}$$

$$\therefore \text{ Volume of the roller} = \frac{1408}{63} - \frac{88}{7} = \frac{616}{63} \text{ cubic ft.}$$

1 cubic ft. of iron weighs 486 lbs.

$$\therefore \frac{616}{63} \text{ cubic ft. of iron weigh } \frac{\overset{88}{\cancel{616}}}{\underset{7}{\cancel{63}}} \times \overset{54}{\cancel{486}} = 4752 \text{ lb.}$$

Ex. 11. The area of the base of a right circular cylinder is 1000 sq. in., and the volume is 5 cubic ft. Find the area of the curved surface.

$$\text{Height of the cylinder} = 5 \times \frac{1728}{1000} = 8{\cdot}64 \text{ in.}$$

$$\text{Radius of the base} = \sqrt{\frac{1000\times 7}{22}} = 17{\cdot}837 \text{ in.}$$

$$\text{And circumference} = 2\times 17{\cdot}837\times\frac{22}{7}.$$

$$\therefore \text{ Area of surface} = 2\times \overset{2\cdot548}{\cancel{17\cdot837}}\times\frac{22}{\cancel{7}}\times 8{\cdot}64$$

$$= 968{\cdot}64768 \text{ sq. in.}$$

Ex. 12. Find the volume and surface of a circular ring whose inner and outer diameters are 3 ft. and 5 ft.

The diameter of cross section $= \frac{1}{2}(5-3) = 1$ ft.

$$\text{Area of cross section} = (\tfrac{1}{2})^2\times\frac{22}{7} = \frac{11}{14} \text{ sq. ft.} \quad \text{(i)}$$

$$\text{circumference of cross section} = 1\times\frac{22}{7} = \frac{22}{7} \text{ ft.} \quad \text{(ii)}$$

$$\text{outer circumference} = 5\times\frac{22}{7} = \frac{110}{7} \text{ ft.}$$

$$\text{inner circumference} = 3\times\frac{22}{7} = \frac{66}{7} \text{ ft.}$$

Mean of circumferences or

$$\text{length of ring} = \tfrac{1}{2}\frac{(110+66)}{7} = \frac{88}{7} \text{ ft.} \quad \text{(iii)}$$

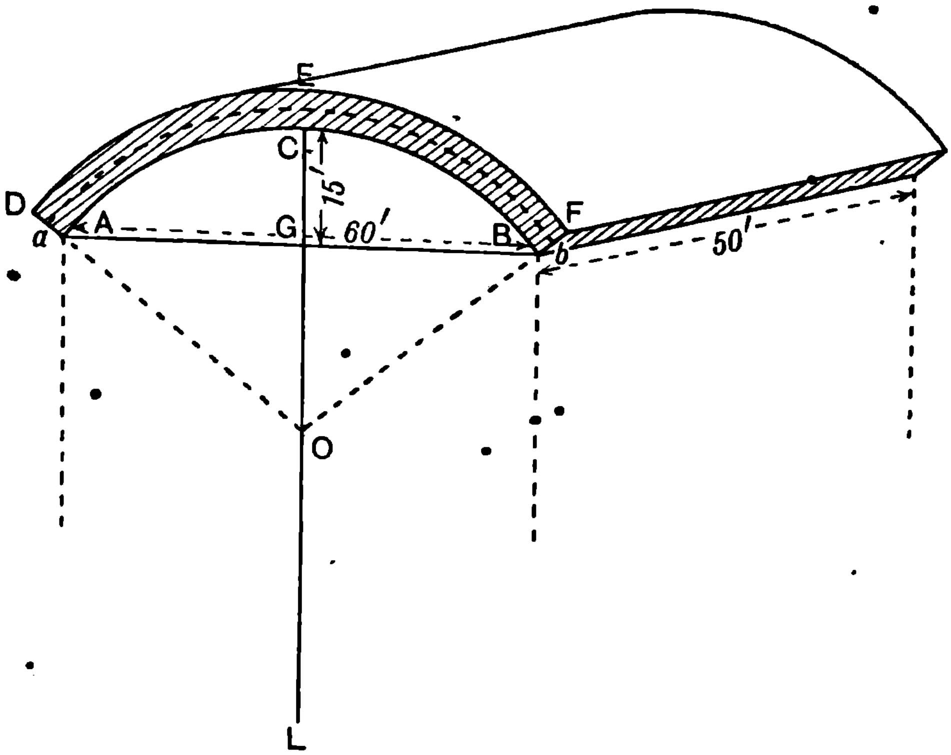
$$\text{Volume of ring} = \text{(i)}\times\text{(iii)} = \frac{11}{\underset{7}{\cancel{14}}}\times\frac{\overset{44}{\cancel{88}}}{7} = \frac{484}{49} = 9{\cdot}87 \text{ cubic ft.}$$

$$\text{Surface} = \text{(iii)}\times\text{(ii)} = \frac{88}{7}\times\frac{22}{7} = \frac{1936}{49} = 39{\cdot}51 \text{ sq. ft.}$$

Ex. 13. Find the number of cubic feet of arched masonry in a bridge whose dimensions are as follows: Span 60 ft., rise 15 ft., depth of masonry 4 ft., and length from face to face 50 ft.

In the diagram ACB is the arch

AB = 60 ft.
CG = 15 ft.



AD = EC = FB = 4 ft.
OC is the radius, and
LC is the diameter.

$$AG^2 = CG.LG$$
$$\text{or } 30^2 = 15\,LG$$
$$\therefore\ LG = \frac{30^2}{15} = 60$$

$$r = \frac{60+15}{2} = 37{\cdot}5$$

Chord AC of the half arc $= \sqrt{30^2+15^2} = 33{\cdot}541$

Arc ACB $= \frac{1}{3}(8\times 33{\cdot}541-60) = \dfrac{208{\cdot}328}{3}$

Midlength of the arch as shown by the dotted line ab =
(Since OA : oa = arc ACB : ab
where OA = 37·5 and oa = 39·5)

$$\frac{208{\cdot}328\times 39{\cdot}5}{3\times 37{\cdot}5}$$

Volume of the arch $= \dfrac{208{\cdot}328\times 39{\cdot}5\times 4\times 50}{3\times 37{\cdot}5} = 14629$, nearly.

Exercise 11

Cube and Cuboids

1. Find the volume and surface of the cubes whose edges measure : (i) 8 ft. (ii) 5 ft. 6 in. (iii) 2 yd. 1 ft. 7 in.

2. Find the cost of painting the surface of a cube whose edge is 7 ft. at the rate of a shilling a square yard.

3. Find the volume and surface of the cuboids having the following dimensions :

(i) Length 7 ft., breadth 6 ft., height 4 ft. 6 in.
(ii) Length 8 ft. 4 in., breadth 7 ft. 2 in., height 3 ft. 3 in.
(iii) Length 3 yd. 2 ft. 1 in., breadth 2 yd. 1 ft. 8 in., height 2 ft. 6 in.

4. Find the value of a rectangular block of teakwood measuring 10 ft. by 1 ft. 5 in. and 10 in. thick, at Rs. 6 per cubic foot.

5. How many gallons are contained in a cubical vessel which measures internally 4 ft. in length, breadth, and depth, supposing 1 cubic ft. of water weighs 6·25 gallons.

6. A rectangular tank is 16 ft. long, 8 ft. wide, and 7 ft. deep. How many tons of water will it hold ? (1 cubic ft. of water weighs 1000 oz.)

7. How long will it take to dig a trench 160 yd. long, 16 ft. wide and 14 ft. deep, if 30 tons of earth are removed in a day ? (1 cubic ft. of earth weighs $92\frac{1}{2}$ lb.)

8. A brick with mortar occupies a space of 10 in. long, 5 in. broad, and 3 in. high. How many bricks will be required for a wall 80 ft. long, 16 ft. high, and 15 in. thick ?

9. A rectangular block of metal whose dimensions are 1 ft. 6 in., 1 ft., and 10 in., is thrown into a cistern partly filled with water. If the cistern stands on a base 2 ft. 6 in. by 1 ft. 4 in., and the block is completely immersed, how high will it cause the surface of the water to rise ?

10. A tank, the area of whose base is $9\frac{1}{3}$ sq. ft., contains 60 gallons, of water. Find its depth. (1 gallon = $277\frac{1}{4}$ cubic in.)

11. What number of 4 in. cubes can be cut from a 1 ft. cube ?

12. The dimensions of a dormitory are 140 ft. long, 18 ft. broad, and 12 ft. high. How many soldiers can be accommodated in it, if each is allowed 520 cubic ft. of air?

13. The whole surface of a rectangular solid contains 1224 sq. ft., and the four vertical faces together contains 744 sq. ft. If the height is 12 ft., find the length and breadth.

14. A closed box is made of wood of uniform thickness. Its external dimensions are 11 in., 9 in., and 7 in., and its inner surface is 286 sq. in. Find the thickness of the wood.

15. A rectangular room is 30 ft. long, 20 ft. broad, and 15 ft. high. Find the length of a diagonal drawn from a corner of the floor to the opposite corner of the ceiling.

16. A ship's hold is 102 ft. long, 40 ft. broad, and 5 ft. deep. How many bales of jute each 3 ft. 6 in. long, 2 ft. 3 in. broad, and 2 ft. 6 in. deep, can be stowed into it, leaving a gangway 4 ft. broad?

17. The outside measurements of a rectangular box without the lid are, length 2 ft. 9 in., breadth 2 ft. 3 in., depth 1 ft. 10 in. Find its weight if the material used is $\frac{1}{2}$ in. thick and weighs 40 lb. per cubic ft.

18. A log of timber has a rectangular section 18 in. by 15 in. The length is 10 ft. Find what length remains when $2\frac{1}{2}$ cubic ft. have been cut off one end.

19. A galvanised iron cistern (open at the top) measures externally 2 ft. 8 in. long, 1 ft. 9 in. broad, and 1 ft. $4\frac{1}{2}$ in. deep. If the metal is $\frac{1}{2}$ in. thick, find the weight of the cistern. Find also its total weight when filled with water. (1 cubic ft. of galvanised iron weighs 7215 oz. and 1 cubic ft. of water weighs 1000 oz.)

20. Find the surface and volume of a cube, in which the diagonal of each face is 2 ft. 6 in.

21. Find the surface and volume of a cube whose diagonal is 1 ft. 3 in.

22. The diagonal of a rectangular solid is 29 in. and its volume is 4032 cubic in. If the thickness is 1 ft., find the length and breadth.

23. The diagonal of a rectangular solid is 37 in. and the whole surface is 2352 sq. in. Show that the sum of the edges is 61 in.

24. Find the edge of a cube equal in volume to a rectangular solid whose dimensions are 6 ft. 3 in., 2 ft. 6 in., and 1 ft.

25. Find the surface of a cube whose volume is 4 cubic ft. 1088 cubic in.

26. Find the dimensions of a cubical cistern which contains 800 gallons of water. (1 cubic ft. of water weighs $6\frac{1}{4}$ gallons.)

Examination Questions

27. Three cubes of metal whose edges are 3, 4, and 5 in. respectively are melted and formed into a single cube: if there be no waste in the process, show that the edge of the new cube will be 6 in.

28. Find the length of the longest rod that can be placed in a room 30 ft. long, 24 ft. broad, and 18 ft. high.

29. Water is distributed to a town of 50,000 inhabitants from a reservoir consisting of three compartments, 200 ft. by 100 ft., with vertical sides, and 12 ft. depth of water. The allowance is 15 gallons per head per day. How many days' supply will the reservoir hold?

30. A box without a lid is made of wood an inch thick; the external length, breadth and height of the box are 2 ft. 10 in., 2 ft. 5 in., and 1 ft. 7 in. respectively. Find what volume the box will hold, and the number of cubic inches of wood.

31. The external length, breadth and height of a closed rectangular wooden box are 18 in., 10 in., and 6 in., respectively, and the thickness of the wood is $\frac{1}{2}$ in. When the box is empty it weighs 15 lb. and when filled with sand 100 lb. Find the weight of a cubic inch of wood and of a cubic inch of sand.

32. The diagonal of a cube is 30 in. What is the solid content?

33. A schoolroom is to be built to accommodate 70 children, so as to allow $8\frac{1}{2}$ sq. ft. of floor and $110\frac{1}{2}$ cubic ft. of space for each child. If the room be 34 ft. long, what must be its breadth and height?

34. Taking the dimensions of a brick to be $9'' \times 4\frac{1}{2}'' \times 3''$, find the number required to build a storeroom 14 ft. high and $22' \times 15'$, the walls being 2 ft. thick and the room being provided with a doorway $8' \times 3\frac{1}{2}'$ and two windows $3' \times 2'$.

35. The three conterminous edges of a rectangular solid are 36, 75, and 80 in., respectively. Find the edge of a cube which will be of the same capacity.

36. A cubic foot of gold is extended by hammering so as to cover an area of six acres. Find the thickness of the gold in decimals of an inch, correct to the first two significant figures.

37. A rectangular bath is 14 ft. long, 9 ft. wide, and 4 ft. deep. How much deeper must it be made to hold 180 gallons more ?

38. A reservoir contains 3217428 cubic ft. of water ; its depth is one third of its length, and its breadth is half the difference between the length and one third of the depth. Find the dimensions.

39. A box without a lid measures externally 4 ft. long, 3 ft. wide, and 2 ft. deep ; the material has a uniform thickness of $\frac{3}{4}$ in. If the wood cost 7*s*. 9*d*. per cubic foot, and the making one tenth of the material, find to the nearest penny the cost of the box.

40. A rectangular solid is 13 ft. long, $3\frac{1}{2}$ ft. broad, and 2 ft. high. Find the length of its diagonal, and also the area of a plane passing through two opposite edges of $3\frac{1}{2}$ ft.

41. Find the least length of wall that will be required to enclose a space of 1200 sq. yd. by the side of an existing wall so that only three sides require to be walled up. If the average sectional area of the wall be 18 sq. ft., find the cost of building it with stones 18 in. by 9 in. by $4\frac{1}{2}$ in. at Rs. 80 per 1000 stones.

Prism, Cylinder and Ring

42. The base of a right prism, 1 ft. 8 in. high, is an equilateral triangle on a side of 1 ft. Find the volume and surface area.

43. Find the weight of a right prism of silver 5 in. long, the ends being triangles whose sides are 2·6 in., 2·5 in., and 1·7 in. (One cubic inch of silver weighs 6·08 oz.)

44. A right prism, 5 in. high, stands upon a quadrilateral base ABCD. Find its volume and whole surface, if AB = 7 in., BC = 5 in., CD = 4 in., DA = 4 in., and the angles at A and D are right angles.

45. Find the volume of a right prism 6 ft. long whose ends are equilateral triangles 10 in. sides.

46. Find the volume and vertical surface of a hexagonal column 12 ft. high whose base is a regular hexagon of 10 in. sides.

47. The base of a prism is a trapezium whose parallel sides are 19 ft. and 11 ft., the distance between them is 9 ft. Find the volume of the prism if its height is 12 ft.

48. The base of a prism is a right angled triangle whose hypotenuse is 2 ft. 10 in. If the height is 1 ft. and the sum of the rectangular faces 6 sq. ft. 96 sq. in., find the other sides of the base.

49. Find the volume of a prism of height 6 in., whose cross section is a regular hexagon of side 1·2 in.

50. Find the volume and curved surface of the following cylinders having given:

(i) Radius of the base 7 in., height 5 in.
(ii) Diameter of the base 3 ft. 6 in., height 2 ft. 6 in.
(iii) Radius of the base $3\frac{1}{2}$ in., height 8 in.
(iv) Diameter of the base 4 ft. 1 in. and the height 3 ft. 4 in.

51. The radius of the base of a cylinder is 5 in. and its curved surface is 440 sq. in. Find its height.

52. The diameter of a cylinder is 1 ft. 2 in. and its volume is 1 cubic ft. 1352 cubic in. Find its whole surface.

53. What is the cubical content of a well 3 ft. 6 in. in diameter and 42 ft. deep.

54. How many coins $\frac{3}{4}$ of an inch diameter and $\frac{1}{8}$ of an inch thick must be melted down to form a rectangular solid whose dimensions are 5 in., 4 in., and 3 in.

55. Find the cost of digging a well 3 ft. in diameter and 40 ft. deep at the rate of Rs. 20 per 100 cubic ft.

56. A cube of iron whose edge is 10 in. is wholly immersed in a cylindrical tub partly filled with water, whose diameter is 18 in. How much will it raise the water in the tub ?

57. A cubic inch of gold is drawn into a wire 1000 yards long. Find to the nearest thousandth of an inch the diameter of the wire.

58. How many gallons of water flow through a pipe in 20 minutes if the bore of the pipe is 2 in. and if the water flows at the rate of 4 miles per hour ?

59. A cast iron pipe has an internal diameter of $3\frac{1}{2}$ in., and an external diameter of 4 in. Calculate the weight of a length 8 ft.

of this pipe. (Cast iron weighs 7·2 times heavier than water, and 1 cubic ft. of water weighs 1000 oz.)

60. A cylindrical boiler has a height of 6 ft. and a diameter of base 3 ft. Water is pouring into it from a pipe, diameter of section 2 in., the water running in the pipe at 12 miles per hour. How long will it take to fill the boiler ?

61. From a prism whose length is 10 in., and whose transverse section is a regular hexagon on a side of 8 in., the greatest possible cylinder is cut having the same axis as the prism. Find the surface and volume of the cylinder.

62. A circular shaft is 80 ft. deep and 3 ft. 4 in. in diameter. Find the cost of sinking it at the rate of 8 annas per cubic foot.

63. What is the volume of a disc $1\frac{1}{4}$ in. thick and 10 in. diameter, with a hole at the centre whose diameter is 6 in.?

64. A right circular cylinder whose length is 1 ft., and the radius of whose base is 6 in., is cut into two segments by a plane at right angles to the base and at a distance of $3\sqrt{3}$ in. from the axis. Find the volume of the smaller segment. ($\pi = 3{\cdot}1416$.)

65. Find to the nearest ton what weight of stone will be required to line a semi-cylindrical tunnel 30 ft. in internal diameter and 360 ft. long. The lining is to be 15 in. thick and 4 per cent of the volume is to be deducted for cement. One cubic foot of stone weighs 170 lb.

66. Find (to the nearest tenth of a foot) what length of piping $1\frac{1}{2}$ in. in internal diameter and $\frac{1}{4}$ in. thick, could be made from a cubical block of lead whose edge is 1 ft. And find the weight of 21 ft. of such piping, if a cubic foot of lead weighs 709·1 lb.

67. The whole surface (inner and outer) of a cylindrical tube is 264 sq. in.; if its length is 6 in. and its external radius 4 in., find its thickness.

68. The length of a cylindrical ring is 45 in. and the diameter of the cross section is $2\frac{1}{2}$ in. Find the volume.

69. The radius of the inner circumference of a cylindrical ring is 9 in. and the diameter of cross section is $3\frac{1}{4}$ in. Find the volume.

70. The diameters of the outer and inner circumferences of a cylindrical ring are $10\frac{3}{4}$ in., and $9\frac{1}{2}$ in. respectively. Find the volume and surface.

71. The mean diameter of a cylindrical ring is 1·75 in. and its volume is ·3388 cubic in. Find the diameter of its cross section.

72. The volume of a circular ring is 81·312 cubic in. and the diameter of its cross section is 1·4 in. Find the length of the ring, and its inner and outer diameters.

73. The area of the surface of a cylindrical ring is $214\frac{11}{40}$ sq. in. and the diameter of the cross section is $1\frac{1}{2}$ in. Find the radius of the inner circumference.

Examination Questions

74. A vessel in the shape of a prism on a regular hexagonal base, whose side is 4 in., is filled with liquid. Find to three decimal places of an inch how much the liquid will sink if half a pint is taken away.

75. A subway is to be constructed beneath a railway station from one platform to another, and the horizontal portion of the tunnel, 20 yd. long, is to have its cross section a rectangle surmounted by a semicircle, and its sides and top are to be lined with brick. The total height and breadth, exclusive of the bricks, are 3 yd. and $1\frac{1}{3}$ yd. respectively, and the thickness of the bricks is $4\frac{1}{2}$ in. Find the weight in tons of the bricks required for the work, if each brick contains $\frac{9}{16}$ of a cubic foot, and weighs 5 lb.

76. What are the cubic contents of a shaft the mean section of which is a regular hexagon, $2\frac{1}{2}$ ft. in side, and the height 60 ft. ?

77. Give the rules for finding the contents of a prism.

78. Find the contents of an elliptical arch 40 ft. span and 8 ft. rise ; thickness of arch $3\frac{1}{2}$ ft. at haunch and $2\frac{1}{2}$ ft. at crown, width of arch 21 ft.

79. The intrados and extrados of an arch 33 ft. long, 30 ft. span, and 7 ft. 6 in. rise are true semi-ellipses, the thickness of arch at springing is 3 ft. and at the crown 2 ft. Find the volume of the arch.

80. The base of a certain prism is a regular hexagon, every edge of the prism measures 1 ft. Find the volume of the prism.

81. Find the quantity of masonry in the segmental arch of a masonry bridge, whose radius of the intrados or soffit is 20 ft., thickness of arch is 2 ft., length of arch is 30 ft., angle subtended by the arch at the centre is 84°.

82. The section of a canal is 32 ft. wide at the top, 14 ft. wide at the bottom, and 8 ft. deep. How many cubic yards were exca-

vated in a mile of the canal? Also, if the surface of the water be 26 ft. wide, what is its depth?

83. A room 30 ft. wide and 40 ft. long is roofed by an arch having a rise of 4 ft. at the centre. The arch is 2 ft. thick. Find the quantity of masonry in it to the nearest cubic foot.

84. Find the number of cubic feet of arched masonry in a bridge whose dimensions are as follows: span 60 ft., rise 15 ft., depth of masonry 4 ft., and length from face to face 50 ft.

85. A pond whose area is 4 acres is frozen over with ice to the uniform thickness of 6 in. If a cubic foot of ice weighs 896 oz. avoirdupois, find the weight of ice on the pond in tons.

86. Find the number of cubic feet of masonry in an arch whose span is 20 ft., rise 3 ft., length from face to face 30 ft., and depth of voussoir $2\frac{1}{2}$ ft.

87. A hollow column is circular inside and elliptical outside, the axes of the ellipse are $4\frac{1}{2}$ and 5 ft., and the diameter of the circle 4 ft. Find the volume, the column being 30 ft. high.

88. Find the quantity of masonry in a roof arch, and its cost at the rate of Rs. 35 per 100 cubic ft. Dimensions: length of arch 40 ft., span 15 ft., rise 3 ft., and thickness 6 in.

89. What must be the length to the nearest foot of a hospital to accommodate 50 patients? The building to be 24 ft. wide, side walls 20 ft. high, and the rise of the roof, which is gabled, two-sevenths of the span, allowing 1200 cubic ft. of air space to each patient.

90. The span of a bridge is 30 ft., rise to intrados 7 ft. 6 in., thickness of arch 3 ft., length 30 ft. How many cubic feet of masonry does the arch contain, and what would be the cost of constructing it at the rate of Rs. 30 per 100 cubic ft.?

91. Find the quantity of masonry in a bridge arch of 30 ft. span, rise one fourth of span, thickness of arch 3 ft., and length 21 ft., and the cost of constructing the same at Rs. 30 per 100 cubic ft.

92. Find the quantity of earthwork in a section of a bund 100 ft. long, of which the breadth at the top is equal to the height, the inner slope is 3 to 1, and the outer $1\frac{1}{2}$ to 1, and the height of the embankment 15 ft.

93. Find the number of cubic feet of masonry in an arch whose clear span is 20 ft., rise 5 ft., length from face to face 30 ft., and depth of voussoir 18 in.

94. The trunk of a tree is a right circular cylinder 5 ft. in radius and 30 ft. high. Find the volume of the timber which remains when the trunk is trimmed just enough to reduce it to a rectangular parallelopiped on a square base.

95. A cubic foot of brass is drawn into a cylindrical wire $\frac{1}{40}$ of an inch in diameter. This wire is just long enough to go round a circular field. Find approximately the area of the field in acres.

96. A cubic foot of brass is drawn into a wire one-tenth of an inch in diameter. Find its length.

97. A well $7\frac{1}{2}$ ft. inside diameter, is to be sunk 22 ft. deep, with a brick lining of $13\frac{1}{2}$ in. in thickness. Find

(*a*) Excavation of earthwork.

(*b*) Quantity of brickwork.

98. A well is to be dug 5 ft. diameter clear inside, and 36 ft. in depth (excluding the curb), with brick lining of 9 in. in thickness. Find

(*a*) Excavation of earthwork.

(*b*) Quantity of brickwork.

99. From a cylindrical tank $4\frac{1}{2}$ ft. in diameter, water is drawn off at the rate of 110 gallons per hour. Find (to the tenth of an inch) by how much the surface would be lowered in 27 minutes. ($\pi = 3{\cdot}1416$, and 1 gallon $= 277{\cdot}25$ cubic in.)

100. Prove that the volume of material in a hollow cylinder is equal to $\pi h[r^2-(r-k)^2]$, and explain the meaning of the various symbols.

101. A railing is to be constructed of cylindrical posts and two rows of rectangular rails. If the posts be 6 ft. long, 6 in. in diameter and separated by distances of 7 ft.; also if the cross section of the rails be a rectangle 6 in. by 1 in., find the number of cubic feet of timber required for a railing 1 mile long.

102. An iron pipe is 3 in. in bore, $\frac{1}{2}$ in. thick, and 20 ft. long. Find its weight, supposing that a cubic inch of iron weighs $4{\cdot}526$ oz.

103. The trunk of a tree is right circular cylinder, 3 ft. in diameter and 20 ft. high. Find the volume of the timber which remains when the trunk is trimmed just enough to reduce it to a rectangular parallelopiped on a square base.

104. Find how many pieces of money $\frac{3}{4}$ in. in diameter and $\frac{1}{8}$ in. thick, must be melted down in order to form a cube whose edge is 3 in. long.

105. A well is to be dug 5 ft. inside diameter, and 30 ft. in depth. Find the quantity of earth to be excavated, and the quantity of brickwork required for a lining of 10 in. in thickness.

106. A hollow circular cylinder of cast iron is 31·43 ft. in circumference and 9 ft. 9·5 in. in diameter inside. Find its thickness. (π=3·14159.)

107. A cubic foot of brass is to be drawn into a cylindrical wire one fortieth of an inch in diameter. What will be the length of the wire ?

108. Find the weight of a cast iron pipe whose length is 9 ft., the bore 3 in., and the thickness of the metal 1 in. A cubic inch of cast iron weighs $\frac{1}{4}$ lb.

109. If 1 mile length of copper wire weighs 1 cwt., find the area of a section, copper being 8·96 times as heavy as water, and 1 cubic ft. of water weighing 1000 oz. avoirdupois.

110. Find how many gallons of water can be held in a leathern hose 2 in. in bore and 40 ft. long.

111. A roller is wanted which must be $3\frac{1}{2}$ ft. in length and weigh 10 maunds. It is to be of freestone of the specific gravity 2·5. What must be its diameter (1 seer = 2 lb.).

112. The length of an iron cylindrical vessel with closed ends is 4 ft., its outside circumference is 40 in., and the thickness of the metal 1 in. Find the entire weight when the cylinder is filled with water, iron being $7\frac{1}{9}$ times heavier than water, and water weighing 1000 oz. per cubic foot.

113. The internal depth and the diameter of a hollow cylinder are respectively 4 ft. $2\frac{1}{2}$ in. and 8 in. A solid cylinder of the same depth and $6\frac{1}{4}$ in. diameter, stands inside it. How many gallons of water can be poured into the remaining space if a gallon contains 277·75 cubic in., and the area of a circle is $\frac{11}{14}$ of the square of its diameter.

114. Find the cubic inches of material in a cylindrical tube, the radius of the outer surface being 10 in., the thickness 2 in., and the height 9 in.

115. The radius of the inner surface of a leaden pipe is $1\frac{1}{2}$ in., and the radius of the outer surface is $1\frac{9}{16}$ in. If the pipe be melted and formed into a solid cylinder of the same length as before, find the radius.

116. A square hole 2 in. wide is cut through a solid cylinder of which the radius $\sqrt{2}$ in., so that the axis of the hole cuts at right angles the axis of the cylinder. Find how much of the material is cut out. ($\pi = 3 \cdot 1416$.)

117. Find the solid contents of a cylindrical ring, whose thickness is 9 and inner diameter 32.

118. The inner diameter of a cylindrical ring is 2·5 in., and the outer diameter 3·8 in. Find the volume of the ring and the weight at the rate of 11000 oz. to the cubic foot.

119. A cylindrical ring whose mean diameter is 18 in. weighs $4033\frac{1}{3}$ oz. Find the radius of the transverse circular section if 240 cubic in. of the substance of which it is made weighs 1000 oz.

120. The volume of a cylindrical ring is 800 cubic in., the radius of cross section is 2 in. Find the length of the ring.

121. A carriage drive is to be made round the outside of a circular park whose radius is 585 ft.; the metalling is to be 30 ft. wide and 9 in. deep. What will it cost at Rs. 6 per 100 cubic ft. ?

122. A circular entrenchment 54 yd. in diameter is to be surrounded by a ditch 6 yd. wide at top and 4 yd. wide at bottom, and 5 yd. deep. Find the number of cubic ft. to be excavated.

123. The volume of a cylindrical ring is 100 cubic in., and the length 20 in. Find the inner diameter.

124. If there are 277·2738 cubic in. in a gallon of water how many tanks each containing 1000 gallons would be completely filled by a rainfall of 1·25 in. upon a field 513·47 acres in area ?

125. Water flows from a tank through a circular pipe at the rate of 30 yd. per minute. If the pipe is 7 in. in diameter and the tank is rectangular in shape, 40 yd. long by 25 yd. 2 ft. broad, how long will it be before the level of the water falls 3 in.

126. Find the weight of an iron pipe 10 ft. long, 2 ft. 6 in. in inside diameter, and $1\frac{1}{2}$ in. thick, the specific gravity of iron being 7·14 and the weight of a cubic foot of water 1000 oz.

127. A swimming bath is 20 yd. long and 8 yd. wide, with steps at one end 1 ft. 6 in. wide and 9 in. deep extending over the whole width of the bath till a depth of 4 ft. 6 in. is reached. Then the bottom slopes down to the other end with an inclination of 1 in 15. Find in gallons the quantity of water the bath can contain when full.

128. A cubic foot of brass is drawn into wire $\frac{1}{16}$ in. in diameter. Find the length of the wire.

129. A sheet of metal $\frac{1}{8}$ in. thick is made into a pipe whose internal diameter is $\frac{1}{2}$ in. This pipe is placed round a cylinder 1 ft. in radius. Find how many cubic inches of water it will contain and how many cubic inches of metal are required to make it.

130. A well is to be dug 5 ft. in diameter clear inside, and 36 ft. in depth (excluding the curb) with a brick lining of 9 in. thickness. Find plastering in square feet of exposed surface.

131. The whole surface of a cylindrical tube is 264 sq. in. If its length is 5 in. and its external radius 4 in. find its thickness.

132. The same number expresses the solidity and convex surface of a cylinder. What is its diameter?

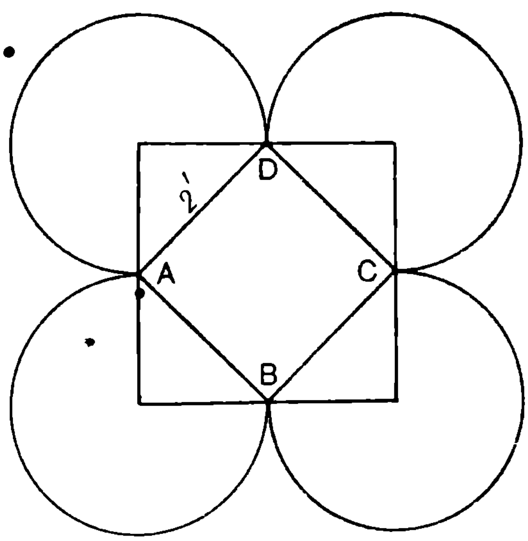
133. What is the proportion between the height of a cylinder and the diameter of its base when the curved surface is equal in area to the two ends?

134. Investigate the area of the surface of a cylindrical ring.

135. The radius of the inner boundary of a ring is 14 in., the area of the surface of the ring is 100 sq. in. Find the radius of the outer boundary.

136. The area of the surface of a ring is 120 sq. in., the radius of the cross section is 1 in. Find the length of the ring.

137. A sheet of metal $\frac{1}{8}$ in. thick is made into a pipe whose internal diameter is half an inch. This pipe is placed round a cylinder 1 ft. in radius. Find the area of the external surface of the pipe.



138. The cross section of a pillar 30 ft. high is as shown, a side of the inner square being 2 ft. and the circular segments touching each other at A, B, C, and D. Find to the nearest rupee the cost of polishing the exposed surface of the pillar at R. 1 per square foot. (The ends of the pillar are not exposed.)

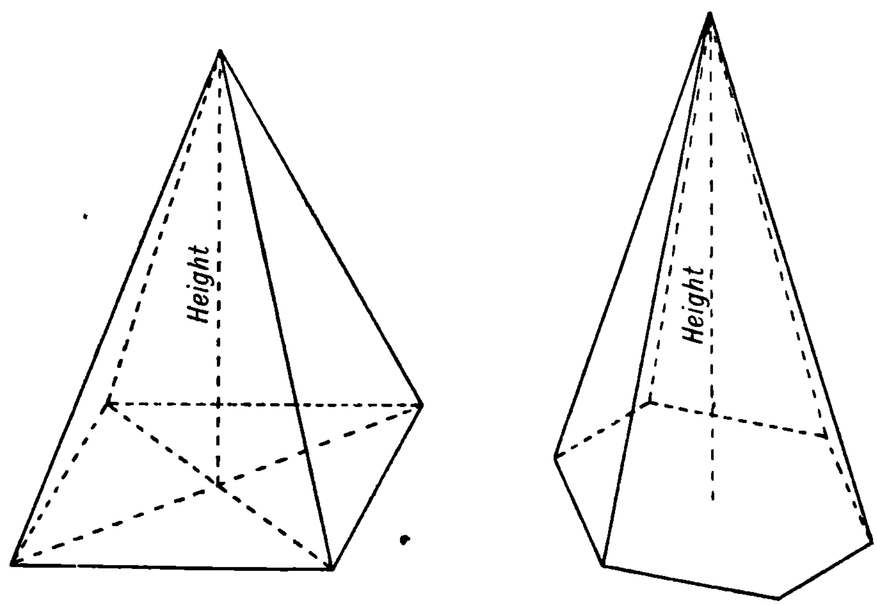
Chapter XV

PYRAMID AND CONE

63. Pyramid* : A pyramid is a solid whose base is a plane rectilinear figure and bounding faces are triangles having a common vertex.

The common vertex of the bounding faces is called the vertex of the pyramid. The perpendicular drawn from the vertex of a pyramid to the base is called the height of the pyramid.

A pyramid is said to be right when a perpendicular dropped from the vertex on the base meets the base at its central point.



The slant height of a right pyramid is the straight line joining the vertex to the middle point of one of the sides of the base.

The sum of the triangular faces is called the slant surface of the pyramid.

In a right pyramid standing on a regular base the triangular faces are all equal to one another.

A tetrahedron is a pyramid on a triangular base ; it is thus contained by four triangles.

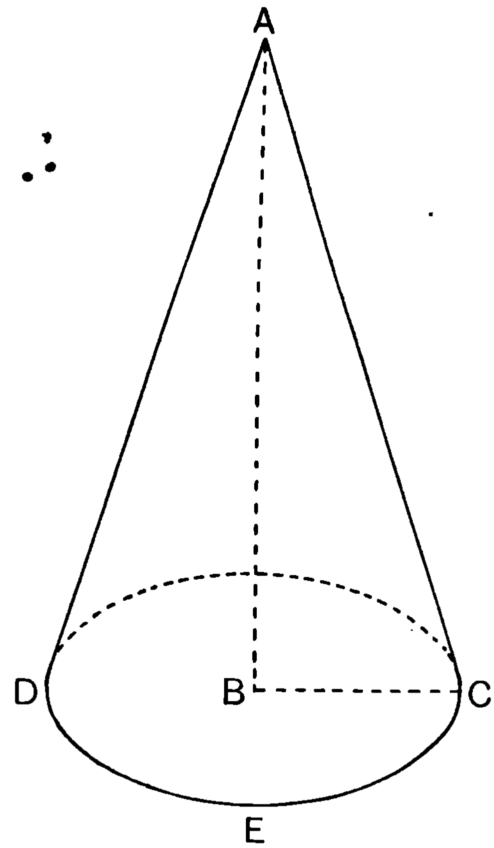
* The term pyramid is taken directly from Greek, but the origin of the word is probably Egyptian, as it was first applied to the Pyramids of Egypt.

64. Cone : A cone is a form of pyramid whose base is a circle. A right circular cone is described by the revolution of a right angled triangle about one of its sides (containing the right angle). In ordinary usage by a cone we mean a right circular cone, and in this sense we shall use the word.

If the right angled triangle ABC revolves about the side AB it describes the cone ADEC. AB is said to be the axis or height of the cone. AC is slant height of the cone.

The volume of pyramid or cone

$$= \frac{\text{Area of the base} \times \text{height.}}{3}$$



The slant surface of a pyramid is equal to the sum of the area of bounding triangles. If the base of the pyramid be a regular rectilinear figure of n number of sides then the area of the slant surface

$$= n \times \text{side of base} \times \tfrac{1}{2} \text{ slant height.}$$

Again, $n \times \text{side} = \text{perimeter of base}$

hence the formula can be put as

$$\text{Perimeter of base} \times \tfrac{1}{2} \text{ slant height.}$$

And this latter formula holds good for the slant surface of a cone, or, in other words, the area of the slant surface of a cone

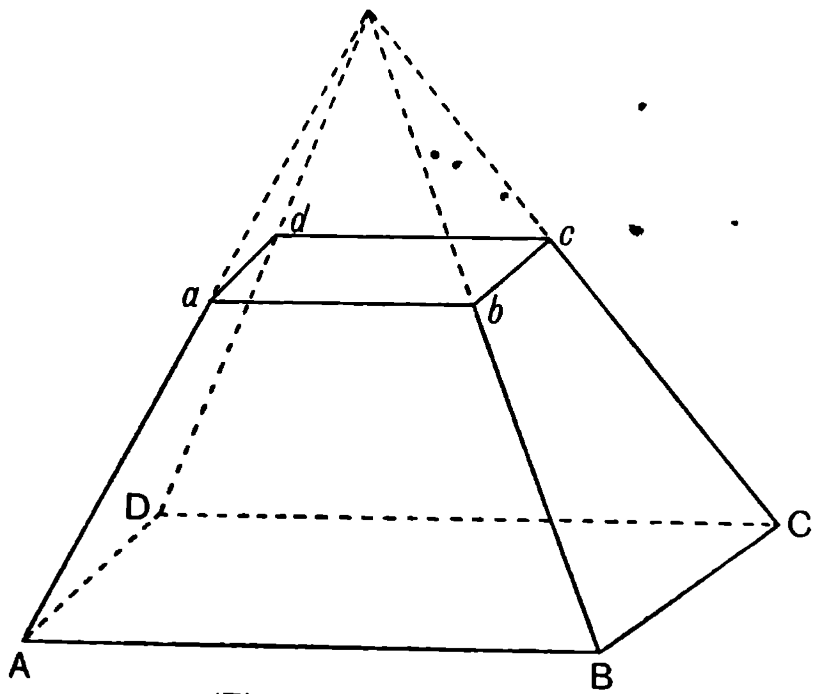
$$= \text{Circumference of the base} \times \tfrac{1}{2} \text{ slant height}$$

$$= \pi \times \text{radius of base} \times \text{ slant height.}$$

65. Frusta *of Pyramid or Cone.*

A frustum* (or slice) of a pyramid or cone is the part contained between the base and a plane parallel to the base.

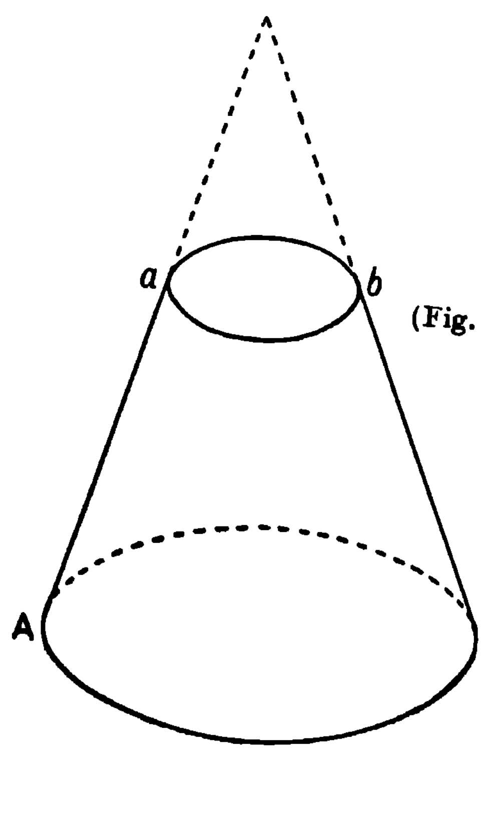
[Note.—A part contained between any two planes both of which are parallel to the base is also a frustum.]



(Fig. i)

In Fig. (i) the part included between the base ABCD and the parallel plane abcd is called the frustum of the pyramid.

In Fig. (ii) the part contained between the base AB and the parallel plane ab is the frustum of the cone.



(Fig. ii)

* Frustum is Latin word meaning "part broken off" derived from *frango*, I break.

The figures ABCD and abcd in Fig. (i), and AB and ab in Fig. (ii) are called the ends of the frustum. The ends of the frustum of a pyramid are similar figures. The ends of the frustum of a cone are circles.

The slant surface of the frustum of a pyramid is made of trapeziums, which are all equal if the pyramid is right and the base ABCD is a regular polygon.

[Note that the portion cut off from a pyramid or a cone to form a frustum is itself a pyramid or a cone.]

The perpendicular distance between the ends of a frustum is called the height or thickness of the frustum. The perpendicular distance between any two corresponding sides of the ends is called the slant height or slant thickness*.

The area of the slant surface

$= \frac{1}{2}$(sum of perimeters of ends) $\times$ slant thickness.

So in the case of the frustum of a cone the area of curved surface

$= \pi$(sum of radii of the ends) $\times$ slant height.

Volume of frustum

$= \frac{1}{3}$ height $\times$ [sum of the area of ends $+$ square root of product of areas of the ends.]

If E^1 and E^2 denote the area of the ends then the formula stands as $\frac{1}{3}$ height $\times [E^1+E^2+\sqrt{E^1 \times E^2}]$

Let the radii of the ends of the frustum of cone be r_1 r_2 then the volume of the frustum of a cone

$$= \frac{1}{3}\text{ height}[\pi r_1^2+\pi r_2^2+\sqrt{\pi r_1^2 \times \pi r_2^2}]$$

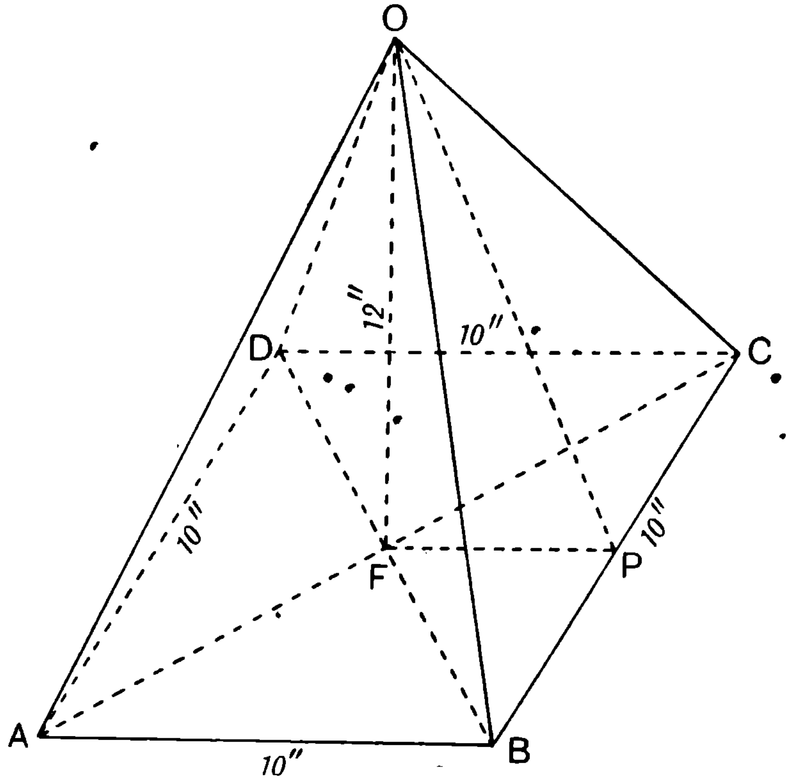
$$= \frac{1}{3}\text{ height}[\pi r_1^2+\pi r_2^2+\pi r_1 \cdot r_2]$$

$$= \frac{\pi}{3}\text{ height}[r_1^2+r_2^2+r_1 \cdot r_2]$$

Examples

Ex. 1. A right pyramid 12 in. high has a square base of 10 in. side. Find the volume and slant surface.

* Here the ends of a cone which are circles are assumed to be rectilinear figures of innumerable sides. The slant height of a frustum is that portion of the slant height of the pyramid or the cone as is left in the frustum.



$$\text{Volume} = \frac{1}{3} \times 10 \times 10 \times 12 = 400 \text{ cubic. in.}$$

$$\text{slant height} = OP = \sqrt{OF^2 + FP^2}$$

$$(FP = \tfrac{1}{2} \text{ side of base} = 5 \text{ in.})$$

$$\therefore \text{ slant height} = \sqrt{12^2 + 5^2} = \sqrt{169}$$

$$= 13 \text{ in.}$$

$$\text{Slant surface} = 4 \times 10 \times \frac{13}{2} = 260 \text{ sq. in.}$$

Ex. 2. Find the volume and surface of right pyramid on a regular hexagonal base; each side of the base is 10 ft. and the height 90 ft.

$$\text{Volume} = \tfrac{1}{3} \text{ height} \times \text{area of the base.}$$

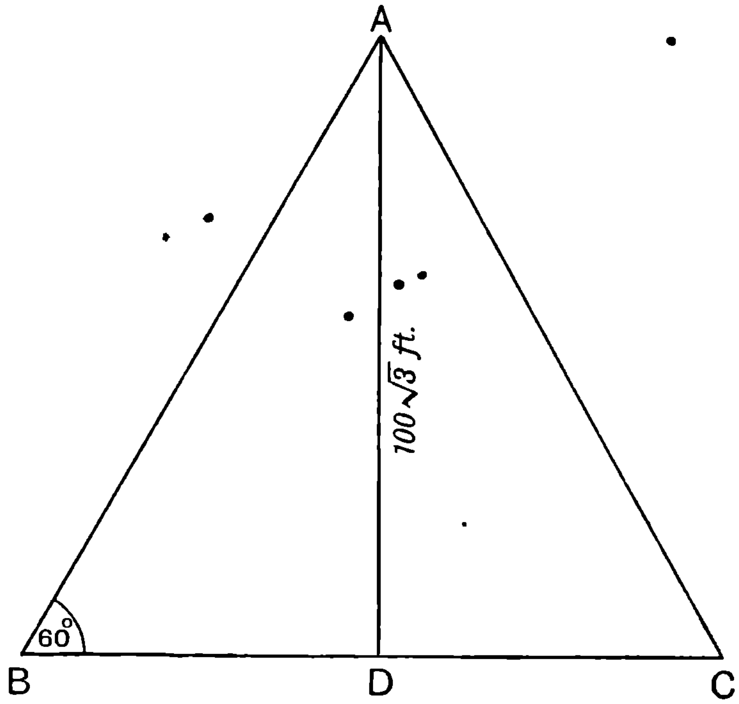
$$= \frac{90}{3} \times \frac{3 \times 10^2 \times \sqrt{3}}{2}$$

$$= 90 \times 10 \times 10 \times \cdot 866 = 7794 \text{ cubic ft.}$$

$$\text{Slant height} = \sqrt{90^2 + (5\sqrt{3})^2} = 90{\cdot}4157 \text{ ft.}$$

$$\text{Slant surface} = 6 \times \frac{10}{2} \times 90{\cdot}4157 = 2712{\cdot}471 \text{ sq. ft.}$$

Ex. 3. A right cone is $100\sqrt{3}$ ft. high and its generating line is inclined at an angle of 60° to the horizon. Find its volume and curved surface.



ABC shows the vertical section of the cone through the axis. BD is the radius of the base and

$$= \frac{100\sqrt{3}}{\sqrt{3}} = 100 \text{ ft.}$$

AB is the slant height and $= 2 \times \text{BD} = 200$ ft.

$$\text{Volume} = \text{Area of the base} \times \frac{\text{height}}{3}$$

$$= \overset{1\cdot0472}{\cancel{3\cdot1416}} \times 100 \times 100 \times \frac{100\sqrt{3}}{\cancel{3}}$$

$$= 1813802\cdot76 \text{ cubic ft.}$$

$$\text{Curved surface} = \pi \times \text{radius of base} \times \text{slant height.}$$

$$= 3\cdot1416 \times 100 \times 200$$

$$= 62832 \text{ sq. ft.}$$

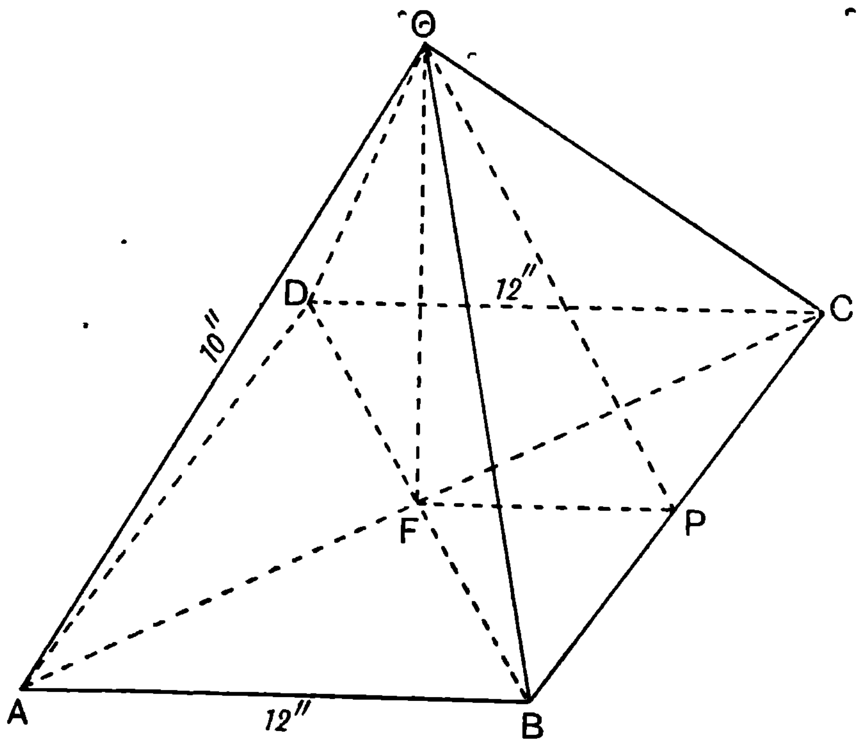
Ex. 4. The base ABCD of a pyramid is a square, each side of which is 12 in. long. Each of the other four edges, OA, OB, OC, OD, is 10 in. long. Find the volume and slant surface of the pyramid.

OP is the slant height and

$$= \sqrt{OB^2 - BP^2}$$

$$\left(BP = \frac{BC}{2} = \frac{12}{2} = 6 \text{ in.}\right)$$

$$\therefore\ OP = \sqrt{10^2 - 6^2} = 8$$



Again,

OF is the height and

$$= \sqrt{OP^2 - FP^2}$$

$$\left(FP = \frac{AB}{2} = 6\right)$$

$$\therefore\ OF = \sqrt{8^2 - 6^2} = \sqrt{28} = 5{\cdot}2915 \text{ in.}$$

$$\text{Volume} = 12 \times 12 \times \frac{5{\cdot}2915}{3} = 253{\cdot}992 \text{ cubic in.}$$

$$\text{Slant surface} = 4 \times 12 \times \frac{8}{2} = 192 \text{ sq. in.}$$

Ex. 5. The perpendicular height of a pyramid is 100 ft., and its base is 120 ft. square; 35 ft. of perpendicular height is removed from the summit. What will be the volume and slant surface of the remainder frustum of the pyramid.

The top end of the frustum will be square and if a side of it be x then, by similar figures:

$$x : 120 = 35 : 100$$

$$\text{or } x = \frac{120 \times 35}{100} = 42$$

$$\text{Volume of frustum} = \frac{65}{3}\left(42^2 + 120^2 + \sqrt{42^2 \times 120^2}\right)$$

$$= \frac{65}{3}(1764 + 14400 + 5040)$$

$$= \frac{65}{3} \times 21204 = 459420 \text{ cubic ft.}$$

The slant height of the pyramid

$$= \sqrt{100^2 + 60^2} = 20\sqrt{5^2 + 3^2} = 20\sqrt{34}$$

If y be the slant height of the frustum, then by similar figures

$$y : 20\sqrt{34} = 65 : 100$$

$$\text{or } y = \frac{20\sqrt{34} \times 65}{100} = 13\sqrt{34}.$$

Hence area of slant surface

$$= \frac{4 \times (42 + 120)}{2} \times 13\sqrt{34}$$

$$= 324 \times 13\sqrt{34} = 324 \times 13 \times 5{\cdot}83095$$

$$= 24559{\cdot}96 \text{ sq. ft.}$$

Ex. 6. The curved surface of a right circular cone is $204\frac{2}{7}$ sq. in., and the height 12 in. Find the radius of the base.

If r be the required radius then curved surface $= \pi r\sqrt{h^2 + r^2}$

$$\text{Here } 204\tfrac{2}{7} = \pi r\sqrt{12^2 + r^2}$$

$$\text{or } \frac{1430 \times 7}{22 \times 7} = r\sqrt{12^2 + r^2}$$

$$\text{or } 65 = r\sqrt{12^2 + r^2}$$

$$\text{or } 65^2 = r^2(144+r^2)$$

$$\text{or } 4225 = 144r^2+r^4$$

$$\text{or } 4225+72^2 = (r^2+72)^2$$

$$\text{or } \sqrt{4225+5184} = r^2+72$$

$$\text{or } 97-72 = r^2$$

$$\therefore r = \sqrt{25} = 5 \text{ in.}$$

$$\text{Radius} = 5 \text{ in.}$$

Ex. 7. Find the curved surface and volume of the frustum of a cone whose slant height is 5 in. and whose circular ends are 8 in. and 6 in. in diameter.

$$\text{Curved surface} = \frac{\pi}{2}(8+6)\times 5$$

$$\frac{22}{7\times 2}\times 14\times 5 = 110 \text{ sq. in.}$$

$$\text{Height of the frustum} = \sqrt{5^2-1^2} = 2\sqrt{6}$$

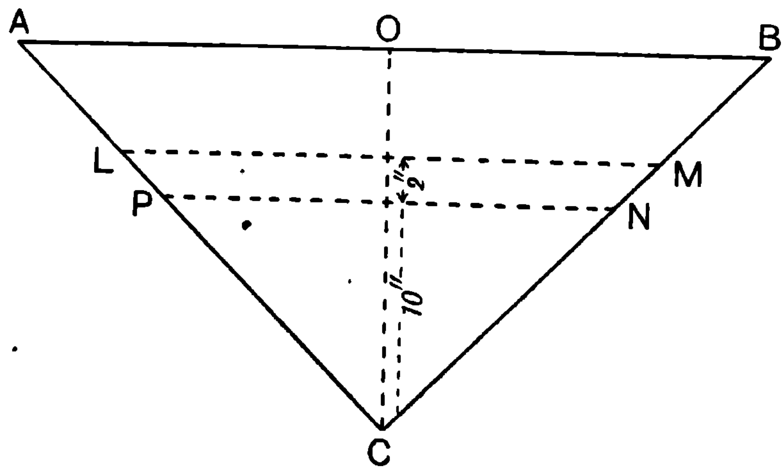
$$\text{Volume} = \frac{\pi}{3}\times 2\sqrt{6}(4^2+3^2+4\times 3)$$

$$= \frac{22}{7\times 3}\times 2\sqrt{6}\times 37$$

$$= 189{\cdot}8938 \text{ cubic in.}$$

Ex. 8. Water is poured into a conical vessel whose angle is 90°, until the surface stands 10 in. above the vertex. How many gallons must now be added to raise the surface by 2 in.?

($\pi = \frac{22}{7}$; one gallon water = 277·25 cubic in.)



The diagram represents a cross-section of the vessel through the axis.

$$\text{Volume of the frustum LMNP} = \frac{\pi}{3}\times 2\,(12^2+10^2+12\times 10)$$

$$= \frac{22}{7\times 3}\times 2\times 364 = \frac{2288}{3} \text{ cubic in.}$$

$$\text{Water required} = \frac{2288}{3}\div 277{\cdot}25 = 2{\cdot}75 \text{ gallons.}$$

Ex. 9. The volume of a frustum of a cone is 407 cubic in., and its thickness is $10\frac{1}{2}$ in. If the diameter of one end is 8 in., find the diameter of the other end. ($\pi = \frac{22}{7}$.)

Let the diameter of the other end be x, then,

$$407 = \frac{22}{7\times 3}\times\frac{21}{2}\left(4^2+\frac{(x)^2}{4}+4\times\frac{x}{2}\right)$$

$$= 11[16+\tfrac{1}{4}x^2+2x]$$

$$\text{or } \frac{407}{11} = 16+\tfrac{1}{4}x^2+2x$$

$$\text{or } 37-16 = \tfrac{1}{4}x^2+2x$$

$$\text{or } 21 = \tfrac{1}{4}x^2+2x$$

$$\text{or } 84 = x^2+8x$$

$$\text{or } 100 = x^2+8x+16$$

$$\text{or } x+4 = 10$$

$$\therefore\ x = 6 \text{ in.}$$

Ex. 10. During a fall of rain a common bucket 12 in. deep was placed out on a level terrace, and at the end of one hour it was found that the water stood in the bucket at a perpendicular height of 4 in. The diameter of the bucket at the mouth and bottom was 9 in. and 3 in. respectively. Find the rate per hour at which the rain was falling.

The figure ABCD represents a vertical mid section of the bucket.

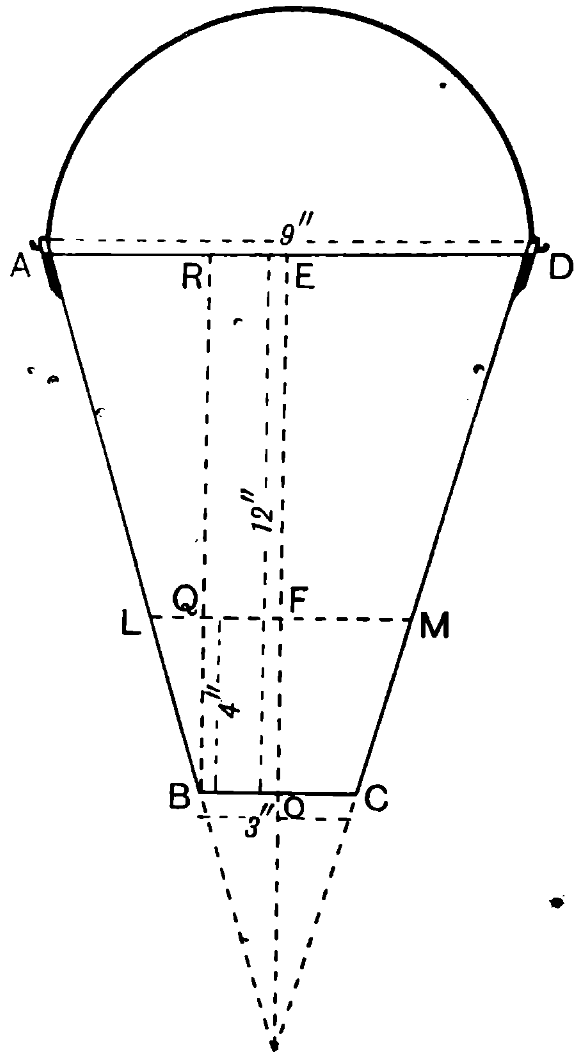
Triangles ABR and LBQ are similar

$\therefore$ LQ : BQ = AR : BR

or LQ : 4 = 3 : 12

$\therefore$ LQ = $\frac{4\times3}{12}$ = 1 in.

and LM = 1+1+3 = 5 in.



Hence volume of water in the bucket

$$= \frac{4\pi}{3}\left[\left(\frac{5}{2}\right)^2+\left(\frac{3}{2}\right)^2+\frac{5}{2}\times\frac{3}{2}\right]$$

$$= \frac{4\pi}{3}\times\frac{49}{4} = \frac{49\pi}{3} \text{ cubic in.}$$

Now consider a cylinder whose cross section is the same as the mouths of the bucket. Such a cylinder will obviously admit the same quantity of rain water in any given time as the bucket.

Hence, if h be the depth of water in such a cylinder after one hour's rainfall, we have

$$\pi\left(\frac{9}{2}\right)^2 h = \frac{49\pi}{3}$$

$$\therefore\ h = \frac{49\pi\times4}{3\times\pi\times81} = \frac{196}{243} = \cdot806 \text{ in.}$$

i.e. the rainfall per hour is ·806 in.

Exercise 12

1. Find the volume of the pyramids in which
   (i) the base is a square of 15 in. and the height is 8 in.
   (ii) the base is a rectangle measuring 12 ft. by 10 ft., and height 32 ft.

(iii) the base is a triangle whose sides are 16 in., 14 in., and 12 in., and the height is 9 in.

(iv) the base is an equilateral triangle of side 1 ft. and height 4 ft.

(v) the base is a square of side 20 in. and the faces are equilateral triangles.

2. Find the volumes of the circular cones in which

(i) radius of the base is 7 in., height 12 in.;

(ii) radius of the base is 3 in., height 14 in.;

(iii) diameter of the base is 1 ft. 9 in., height 1 ft.;

(iv) diameter of the base is 6 ft., slant height 7 ft. 1 in.;

(v) the height is 2 ft., slant height 2 ft. 1 in.

3. Find the weight to the nearest pound of a cone 15 in. high, whose base is 10 in. in diameter, if the material of which it is made weighs 500 lb. per cubic ft.

4. The height of a pyramid standing on a rectangular base is 2 ft. 8 in., and its volume is 5 cubic ft. If the length of the base is 3 ft. 9 in. find its breadth.

5. The generating line of a right cone is inclined at an angle of 60° to the horizon. If the height of the cone measure 15 in., find its volume.

6. A right pyramid stands on a square base containing 300 sq. in. and the perpendicular height of the pyramid is half its slant height. Find its volume.

7. The base of a pyramid 6 in. high, is an equilateral triangle. If the volume is 346·41 cubic in., find the length of a side of the base.

8. Find the radius of the base and the slant height of a cone of which the volume is 3 cubic ft. 96 cubic in., and the height 2 ft. 11 in.

9. A conical vessel 2 ft. 4 in. across the top and $4\frac{4}{5}$ ft. deep, is placed with its axis vertical and vertex downwards. How many gallons of water will it hold, if one cubic ft. of water weighs $6\frac{1}{4}$ gallons?

10. A cone whose vertical angle is 60° is pressed (vertex downwards) into a vessel full of water. When 6 gallons have overflowed, find the depth of the vertex below the surface of the water.
(1 cubic ft. of water = $6\frac{1}{4}$ gallons.)

11. Find the slant surface of a right pyramid one foot high standing on a rectangular base whose length and breadth are 5 ft. 10 in., and 10 in.

12. Find the area of the whole surface of a right pyramid whose base is a regular hexagon of side 10 in., and slant height is 1 ft.

13. Find the cost of polishing the slant surface of a right pyramid 6 ft. 5 in. high, standing on a square base, each side of which measures 6 ft., at the rate of 9*d*. per sq. ft.

14. The slant faces of a right pyramid, standing on a square base are equilateral triangles, each side of which is 16 in. Find the height and the slant surface.

15. A right pyramid stands on a hexagonal base each side of which is 14 in. If each slant edge is 21 in. find the height and slant surface.

16. A pyramid on a square base has four equilateral triangles for its four other faces, each edge being 9 in. Find the slant surface.

17. Find the curved surfaces of the cones of which

(i) the radius of base is 1 ft. 2 in., slant height 2 ft.;

(ii) the radius of base is 2 ft. 3 in., slant height 5 ft.;

(iii) the diameter of base is 1 ft. 10 in., height 5 ft;

(iv) height is 3 ft. 4 in., diameter of base 1 ft. 6 in.

(v) circumference of base is 11 ft., height 6 yd. 4 in. Find the whole surface.

(vi) Slant height 2 ft. 1 in., height 2 ft. Find the whole surface.

18. The curved surface of a right circular cone is 176 sq. in., and the slant height is 8 in. Find the radius of the base.

19. The curved surface of a right circular cone is 550 sq. in., and the height is 2 ft. Find the radius of the base.

20. Find the curved surface of a cone whose height is 20 in., and whose volume is 9240 cubic in.

21. A conical vessel has a diameter at the surface of 30 in., and a depth of 1 ft. Find how long it would take to fill it with water flowing at the rate of 50 ft. per minute from a cylindrical pipe ·6 in. in diameter.

Frusta of Pyramid and Cone

22. The ends of a frustum of a pyramid are squares on sides of 20 in. and 4 in. If the frustum is 15 in. thick, find the slant surface.

23. Find the volume of the frustum of a pyramid, the ends being squares on sides of 8 in. and 6 in., and thickness being 3 in.

24. The ends of a frustum of a pyramid are rectangles, the base measuring 9 in. by 6 in., and the top 2 in. by 2 in. If the thickness is 5 in., find the volume.

25. The ends of a frustum of a pyramid are squares, the lengths of the sides being 8 in. and 1·4 in. respectively; the height is 5·6 in. Find the slant surface.

26. The ends of a frustum of a pyramid are regular hexagons on sides of 6 in. and 4 in. If the thickness of the frustum is 5 in., find its volume.

27. If a block of stone in the form of a frustum of a pyramid with square ends taper from a width of 28 to 14 in. in a length of 18 ft. 9 in. measured perpendicular to the ends, what is the volume?

28. The circumference of one end of a frustum of a cone is 48 in., and of the other end 34 in., and the height of the frustum is 10 in. Find the volume.

29. Find the volume of the frustum of a cone in which

(i) The radii of the ends are 5 in. and 3 in. and thickness 1 ft.

(ii) The ends are 2 ft. 6 in., and 1 ft. 6 in. in diameter, and the thickness is 9 in.

30. The ends of the frustum of a cone are 14 in. and 28 in. in diameter; and the slant height is 10 in. Find the whole surface.

31. The slant height of the frustum of a cone is 5 in. and the ends are 8 in. and 2 in. in diameter. Find the volume.

32. Find the slant surface of the frustum of a cone 1 ft. thick and whose ends are 2 ft. 6 in. and 1 ft. 8 in. in diameter.

33. An open vessel in the form of a frustum of a cone is to be lined with metal which costs 3*s*. 6*d*. per sq. ft. Find the whole cost if the upper and lower diameters of the vessel are 1 ft. 8 in. and 8 in., and its depth is 8 in.

34. The slant height of the frustum of a cone is 2 ft. 1 in. and the diameters of its ends are 2 ft. 4 in. and 1 ft. 2 in. Find its volume.

35. A cone 31·5 in. in height, standing on a base 60 in. in diameter, is cut through by a plane parallel to the base and 21 in. above it. Find the curved surface of the frustum so formed.

36. A conical vessel $7\frac{1}{2}$ in. deep and 20 in. across the top is completely filled with water. If water is now drawn off to lower its level by 6 in., find the surface of the vessel thus exposed.

37. A brick vat is in the form of a frustum of a square pyramid. Each side of the base measures 6 ft. and the slant faces are inclined to the base at an angle of 135°. If the vat is 1 yd. deep, how many gallons of water will it hold ? (1 cubic ft. of water $= 6\frac{1}{4}$ gallons.)

38. A cone 2 ft. in height, standing on a base 1 ft. 8 in. diameter, is cut by a plane parallel to the base, so as to have a frustum whose slant thickness is 1 ft. 1 in. Find the volume of the frustum.

39. Water is poured into a conical vessel whose angle is 60°, until the surface of the water is 6 in. above the vertex. A solid cube of metal is now dropped in and is totally submerged. If the surface of the water rises one-tenth of an inch, find the edge of the cube to two places of decimals of an inch.

40. How many square yards of canvas will be required for a conical tent 9 ft. high, so that a man of 6 ft. can stand upright anywhere within 2 ft. of the central pole ?

41. Find the volume of a cask in the form of a double conical frustum, the ends having a diameter of 2 ft. and the middle 3 ft., the height being 4 ft.

42. Cleopatra's Needle consists approximately of a frustum of a pyramid surmounted by a smaller pyramid. The lower base is $7\frac{1}{2}$ ft. square and the upper base $4\frac{1}{2}$ ft. square ; the height of the frustum is 61 ft. and of the upper pyramid $7\frac{1}{2}$ ft. Calculate the weight to the nearest ton. (1 cubic ft. weighs about 170 lb.)

43. A tumbler is in the form of a frustum of a cone. The diameters of the top and bottom are 3 in. and 2 in. respectively, and the depth is 5 in. Express as a decimal of a pint the amount of liquid the tumbler will hold. (1 gallon = 277·3 cubic in. = 8 pints.)

44. A tent is made in the form of a conic frustum surmounted by a cone. The diameters of the top and bottom of the frustum are 16 ft. and 20 ft. respectively, and the frustum and the cone are each 6 ft. high. Find the cubic feet of air contained in the tent.

45. On a piece of ground 12 ft. square, coal is stacked. The top of the stack is also a square of 4 ft. side. The vertical height is

10 ft. Find the quantity of coal in cubic feet and the total surface exposed.

46. The frustum of a square pyramid is 9 in. thick and the area of one end is nine times the area of the other. If the volume is 624 cubic in., find the dimensions of the ends.

Examination Questions

47. Find the volume of a pyramid when its base is a regular hexagon, each side measuring 6 ft. and height 30 ft.

48. A regular hexagonal pyramid has the perimeter of its base 15 ft. and its altitude 15 ft. Find its volume.

49. The spire of a church is a right pyramid on a regular hexagonal base; each side of the base is 10 ft. and the height is 50 ft. There is a hollow part, which is also a right pyramid on a regular hexagonal base. The height of the hollow part is 45 ft., and each side of the base is 9 ft. Find the number of cubic feet of masonry in the spire.

50. A pyramid on a square base has four equilateral triangles for its four other faces, each edge being 20 ft. Find the volume.

51. Find the number of cubic feet in a regular hexagonal room, each side of which is 20 ft. in length, and the walls 30 ft. high, and which is finished above with a roof in the form of a hexagonal pyramid 15 ft. high.

52. A solid is bounded by four equilateral triangles, a side of each triangle being 12 in. Find the volume.

53. Find the volume of the regular triangular pyramid, a side of its base being 6 ft. and its altitude 60 ft.

54. A pyramid on a square base has four equilateral triangles for its four other faces, each edge being 30 ft. Find the volume.

55. A pyramid has for its base an equilateral triangle of which each side is 2 ft., and its slant edge is 6 ft. Find its solid content.

56. The representative gold pyramid in the international exhibition of 1862 was 10 ft. square at the base and 44 ft. $9\frac{1}{2}$ in. in height. Find the volume in cubic feet; also the weight if 1 cubic in. of gold weighs 10·14502 oz. troy, and the value at 80*s*. per ounce.

57. A right angled triangle, of which the sides are 3 ft. 6 in. and 5 ft. in length is made to turn round on the longer side. Find the volume of the solid thus formed.

58. The section of a right circular cone by a plane through its vertex perpendicular to the base is an equilateral triangle, each side of which is 12 ft. Find the volume of the cone.

59. A cone 3 ft. high and 2 ft. in diameter at the bottom is placed on the ground, and sand is poured over it until a conical heap is formed 5 ft. high and 30 ft. in circumference at the bottom. Find how many cubic feet of sand there are.

60. A right angled triangle, of which the sides are 3 in. and 4 in. in length, is made to turn round on the longer side. Find the volume of the cone thus formed.

61. Find how many gallons are contained in a vessel which is in the form of a right circular cone, the radius of the base being 8 ft. and the slant side 17 ft.

62. Find the solidity of a cone, the diameter of whose base is 3 ft. and its altitude 30 ft.

63. The faces of a pyramid on a square base are equilateral triangles, a side of the base being 120 ft. Find the volume.

64. A right angled triangle, whose remaining angles are 60° and 30° revolves about its hypotenuse, which is 12 in. long. Find the volume of the solid thus described.

65. A conical tent is required to accommodate 5 people; each person must have 16 sq. ft. of space on the ground, and 100 cubic ft. of air to breathe. Give the vertical height, slant height, and width of the tent.

66. A piece of tin having the form of a quadrant of a circle is rolled up so as to form a conical vessel: required its content, when the radius of the quadrant is 10 in.

67. How many gallons of water will result from the melting of a pyramid of ice 3 ft. high and with a hexagonal base of 1 ft. each side, it being given that ice loses 7 per cent of its volume on melting, and that 1 cubic ft. of ice contains $6\frac{1}{2}$ gallons.

68. A pyramidal roof 12 ft. high standing on a rectangular base 18 ft. by 32 ft. is covered with slates which cost 18*s.* 9*d.* per hundred, and each of which has an exposed surface of 12 in. by 9 in. Find the cost.

69. Find the area of the inclined surface of a square pyramid each side of the base being 3 ft., and the slant height 15 ft.

70. It is desired to cover a piece of ground 21 ft. square by a pyramidal tent 14 ft. in perpendicular height. Find the cost of the requisite canvas at 5 annas a square yard.

71. Find the area of the whole surface of a triangular pyramid contained by four equilateral triangles, a side of each being 10 ft.

72. The pyramid of Cheops is 750 ft. square and 450 ft. high. What would it cost to restore the surface to its original splendour with polished granite at the rate of £1 per superficial foot ?

73. A pyramid has for its base an equilateral triangle, of which each side is 2 ft. and its slant edge 6 ft. Find its exposed surface.

74. Find the area of the whole surface of a pyramid on a triangular base, having its other faces equal, each side of the base is 1·45 in., and the slant edge of the pyramid is 2·68 in.

75. The slant edge of a hexagonal spire 75 ft. high is 77 ft. Find the cost of painting at Rs. 4 per 100 superficial feet.

76. Find the cost of canvas required for a single pole tent 12 ft. square with walls 6 ft. high. Roof slopes at an angle of 45°, and projects 3 ft. beyond the walls all round ; canvas cloth, 2 ft. 3 in. wide costs 14 annas per yard.

77. Find the surface of the frustum of a square pyramid, each side of the base or greater end being 3 ft. 4 in., each side of the top or lesser end being 2 ft. 2 in., and each of the edges of the frustum being 10 ft.

78. The ends of a frustum of a pyramid are hexagons with sides of 6 ft. and 4 ft. respectively ; the slant height is 10 ft. Find the surface.

79. The area of the surface of a frustum of a square pyramid is 100 sq. ft., the perimeter of the base is 13 ft. 4 in., and the slant height is 10 ft. Find the area of the top.

80. A frustum of a regular pyramid has square ends ; the edge of the lower end is 10 in., and that of the upper end is 5 in., and the height of the frustum is $7\frac{1}{2}$ in. Find the length of a slant edge of the frustum and the area of the slant faces.

81. How much canvas will make a conical tent 11 ft. in height, and 12 ft. in diameter at the base ?

82. A right angled triangle, of which the sides are 3 in. and 4 in. in length is made to turn round on the longer side. Find the area of the whole surface of the cone thus formed.

83. Calculate to three places of decimals the entire superficial area in square inches of a solid cone, the diameter of its base being 8 in. and its altitude 13 in.

84. The area of the whole surface of a right circular cone is 32 sq. ft. and the slant height is three times the radius of the base. Find the volume of the cone.

85. Find the cost of painting a conical spire 64 ft. in circumference at the base and 108 ft. in slant height, at $7\frac{1}{2}d.$ per square yard.

86. Find the cost of the canvas for 150 conical tents, the height of each being $11\frac{1}{4}$ ft., and the diameter of the base 12 ft., at $5d.$ per square yard.

87. Find what length of canvas $\frac{3}{4}$ yd. wide is required to make a conical tent 12 ft. in diameter and 8 ft. high.

88. How many square yards of canvas will be required for a tent, the wall of which forms a right circular cylinder 10 ft. in diameter, and 8 ft. high, the roof of the tent being a right circular cone with the apex 12 ft. above the ground? The roof does not extend beyond the top of the wall.

89. The ends of a frustum of a pyramid are squares, the lengths of the sides being 20 ft. and 30 ft. respectively. The length of the straight line which joins the middle point of any side of one end with the middle point of the corresponding side of the other end is 13 ft. Find the volume.

90. The ends of a frustum of a pyramid are hexagons with sides of 6 ft. and 4 ft. respectively; the slant height is 10 ft. Find the volume.

91. The ends of a frustum of a pyramid are right angled triangles; the sides containing the right angle of the one end are 2 ft., and 3 ft., the smallest side of the other end is 8 ft., the height of the frustum is 7 ft. Find the volume.

92. The ends of a frustum of a pyramid are equilateral triangles the lengths of the side being 6 ft. and 7 ft., respectively, and the length of the slant edge of the frustum is 9 ft. Find the volume.

93. A tank in the shape of the frustum of an inverted right square pyramid, length of side at bottom 40 ft., and at ground level 120 ft., (the height of the pyramid, if complete, being equal to the side of the base), is to be lined with masonry 2 ft. thick. Find the cost of the masonry at Rs. 2 per cubic foot.

94. What is the solidity of a frustum of a regular hexagonal pyramid the sides of the ends being 4 ft. and 6 ft., and its length 24 ft. ?

95. The height of a frustum of a pyramid is 12·5 in.; its ends are octagons whose sides are 4 in. and 2 in. respectively. Find the volume of the frustum.

96. A cask, in the form of two conic frusta, joined at the base, has the head diameter 20 in., the bung diameter 25 in., and the length 3 ft. 4 in. Find its capacity in gallons. ($277\frac{1}{4}$ cubic in. = 1 gallon.)

97. The circumference of the base of a haystack in the form of a conical frustum surmounted by a cone is 40, the circumference at the eaves is 60, the perpendicular height of the frustum is 15, and that of the cone 16 ft. How many solid yards does the stack contain ?

98. What is the volume of the frustum of a right cone, the area of the two circular ends being 1256·64 and 78·54 in. respectively, 30 in. being the height of the cone before it was truncated ?

99. A silver tumbler is of the shape of a truncated cone. Upper diameter inside 6 in., lower diameter 3 in., height 6 in., thickness of the metal $\frac{1}{8}$ in. Find the weight. (Specific gravity 11·00.)

100. A circular well is 17 ft. 6 in. in diameter and 33 ft. deep. Find the quantity of masonry lining, which is 2 ft. thick at the top and 4 ft. 9 in. at the bottom, the batter being on the rear.

101. The depth of a pail in the form of a frustum of a cone is 10 in., its diameter at the mouth 12 in., and its diameter at the bottom 9 in. Find how often it can be filled from a tank containing 2000 gallons of water. (A gallon = 277·274 cubic in.)

102. In order to drain an acre of land, a tank is dug in the form of a frustum of a cone, the radius of the surface section being 30 yd., and of the bottom 20 yd., and the depth of the tank 15 ft. Assuming that two-fifths of the rainfall does not penetrate the soil, that there is no drainage from the subsoil, and no evaporation, find what the average daily rainfall has been, if, after two months, the tank is two-thirds full. (One month = 30 days.)

103. An iron right circular cone 10 in. high, and whose semi-vertical angle is 30° is cut into two at the midpoint of its height by a plane parallel to the base. The frustum so obtained is drawn into wire, whose diameter is $\frac{1}{16}$ in. Find the length of the wire.

104. The shaft of Pompey's pillar is a single stone of granite. The height is 90 ft., the diameter at one end is 9 ft. and at the other end 7 ft. 6 in. Find the volume.

105. The slant side of the frustum of a right circular cone is 5 ft., and the radii of the ends are 7 ft. and 10 ft. Find the volume.

106. During a fall of rain a common bucket 12 in. deep was placed out on a level terrace, and at the end of one hour it was found that the water stood in the bucket at a perpendicular height of 4 in. The diameter of the bucket at the mouth and bottom was 9 in. and 3 in., respectively. Find the rate per hour at which the rain was falling.

107. A piece of marble in the form of a frustum of a cone has its end diameters $1\frac{1}{2}$ ft. and 4 ft., and its slant side is 8 ft. What will it cost at 12*s*. per cubic foot?

108. A coppersmith proposes to make a flat-bottomed kettle, of the form of a conic frustum, to contain 13·8827 gallons: the depth of the kettle to be 1 ft., and the diameters of the top and bottom to be in the ratio of 5 to 3. What are the diameters?

109. If a cask, which is two equal conic frusta joined together at the bases, has its bung diameter 36 in., its head diameter 20 in., and its length 40 in., how many imperial gallons will it hold?

110. A balcony is supported by six granite columns of the following dimensions: the diameters of each at top and bottom are 2 ft. and $2\frac{1}{2}$ ft. respectively, and length 20 ft. Taking the rate at Rs. 2 per cubic foot, what would be their total cost?

111. A log of wood is in the form of a frustum of a cone; the diameter of the larger end is 16 in., and of the smaller end 12 in., perpendicular height 9 ft. What is its value at Rs. 1–8–0 per cubic foot?

112. The radii of the ends of a frustum are 15 ft. and 24 ft., and the slant height is 12 ft. Find the volume.

113. The lower portion of a haystack is an inverted conic frustum, and the upper part a cone. The greatest height is 25 ft., the greatest circumference is 54 ft., the height of the frustrum 15 ft., and the diameter of the base 15 ft. Find the content in cubic yards.

114. A bucket is in the form of a frustum of a cone. The diameter at the bottom is 1 ft., and at the top 1 ft. 3 in., the depth is 1 ft. 6 in. Find to the nearest pound how much more the bucket weighs when full of water than when empty.

115. A mast is 30 in. in diameter at bottom and 15 in. at top. If the mast contains $137\frac{1}{2}$ cubic ft. of wood, find its height in feet.

116. A bucket is in shape a conical frustum (height = 9 in., diameters of top and bottom surface = 10 in. and $7\frac{1}{2}$ in. respectively). Find how much lower the water will stand in a well, whose diameter is 5 ft., after the bucket has been filled 24 times.

117. Find the number of cubic feet of masonry in a chimney shaft of the following dimensions: diameter of the base 30 ft., and of the top 6 ft., diameter of the flue at the base 3 ft., and at the top 2 ft. The outer face of the chimney is built with a batter of 1 in 20.

118. A well in the form of a cylinder, 4 ft. wide and 12 ft. deep, is emptied by a bucket 21 in. wide at the top, 18 in. wide at the bottom, and 15 in. deep. How many times must the bucket be lowered to empty the well, supposing that on the average when withdrawn it is only eight-ninths full?

119. During a fall of rain a bucket whose upper and lower interior diameters are 15 in. and 8 in., and depth 13 in., was placed out on a flat surface, and after thirty minutes' exposure the depth of water in the bucket was found to be 4 in. What was the rainfall per hour?

120. A tin funnel consists of two parts: one part is conical, the slant side is 6 in., the circumference of one end is 20 in., and of the other end $1\frac{1}{4}$ in., the other part is cylindrical, the circumference being $1\frac{1}{4}$ in., and the length 8 in. Find the number of square inches of tin.

121. The half of a regular hexagon, formed by joining the middle points of two opposite sides of the whole figure, revolves about this line. Determine the whole surface of the solid thus generated, a side of the hexagon being 10 ft.

122. What is the area of the slant surface of a frustum of a right cone, the area of the two circular ends being 1256·64 in., and 78·54 in. respectively, and the vertical height of the frustum 20 in.? ($\pi = 3{\cdot}1416$.)

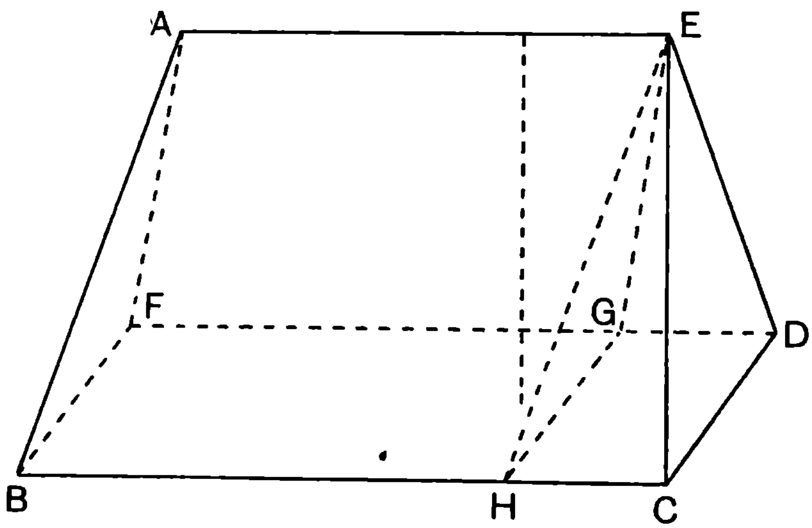
123. A tent is made in the form of a conic frustum, surmounted by a cone. The diameters of the base and the top of the frustum are 14 ft. and 7 ft., its height 8 ft., and the height of the tent 12 ft. Find the quantity of canvas required for it.

CHAPTER XVI

WEDGE AND PRISMOID

66. *Wedge.* A wedge is a solid bounded by five plane surfaces ; the base is a rectangle, the two ends are triangles and the two remaining faces are trapeziums. The line in which the side faces (trapeziums) intersect is called edge of the wedge. The edge of the wedge is parallel to the base.

The height of a wedge is the perpendicular distance between the base and the edge.



In the diagram, BCDF is the base, AE is the edge.

If the edge AE be equal to the length of the base BC, then the faces ABCE and AFDE are parallelograms and the wedge is a triangular prism.

Let a = length of base

e = breadth of base

b = edge.

h = height.

Volume of wedge :

Cut the wedge through E by a plane parallel to ABF. Now the wedge is divided into the prism FBHGEA and pyramid EHCDG.

The volume is therefore = prism FBHGEA + pyramid EHCDG.

But volume of prism = $\frac{1}{2}$ eh × b

and volume of pyramid = $\frac{1}{3}$ h × (a − b) × e

$$\text{Volume of wedge} = \tfrac{1}{2}\,eh \times b + \tfrac{1}{3}\,eh\,(a-b)$$

$$= \frac{ehb}{2} + \frac{eha}{3} - \frac{ehb}{3}$$

$$= \frac{ehb}{6} + \frac{eha}{3}$$

$$= \frac{eh}{6}(2a+b)$$

or briefly :

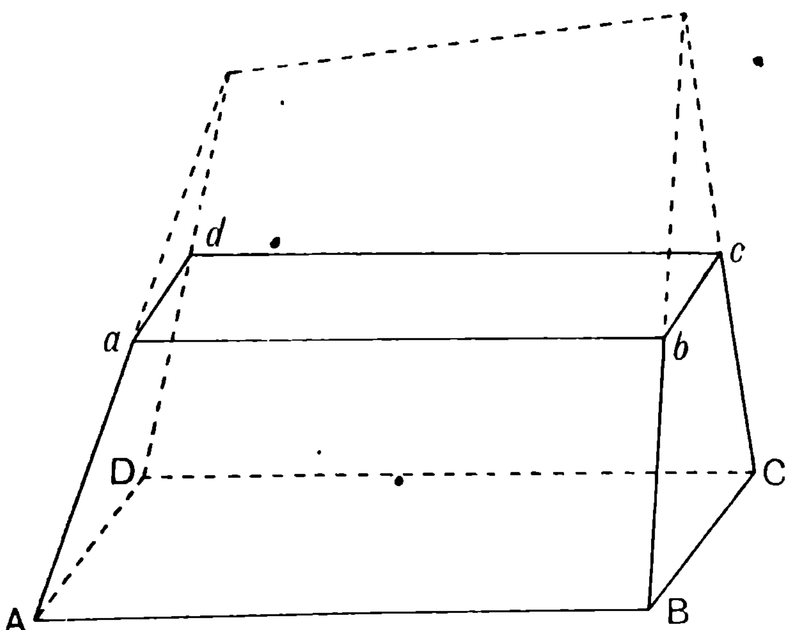
$$\text{Volume of wedge} = \frac{\text{breadth of base} \times \text{height}}{6} \times (2\ \text{length of base} + \text{edge})$$

Surface of wedge :

The surface is found by calculating separately the area of each face. To do this the slant heights of the faces, or the means of finding them, must be given.

[Note : When the edge of a wedge is longer than the length of the base, the same formula may be applied for finding the volume.]

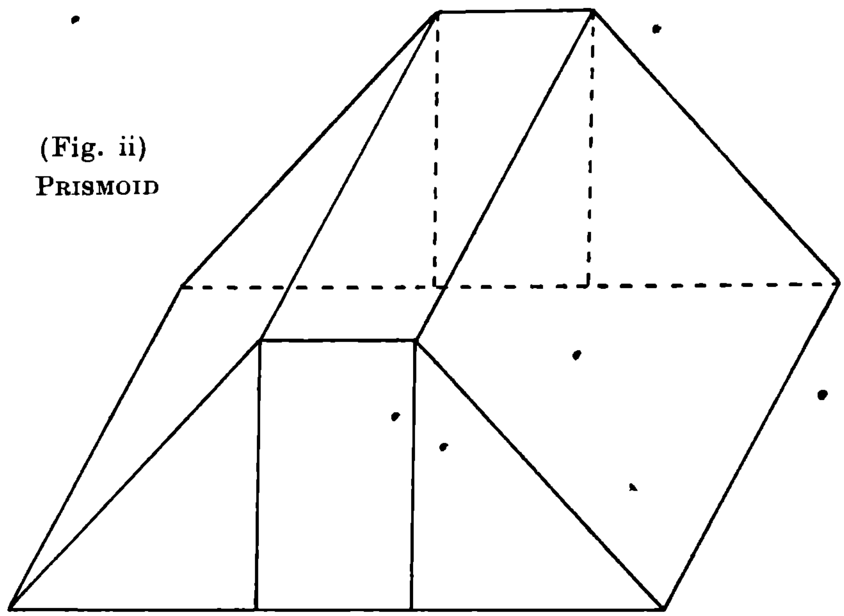
67. The frustum of a wedge is the part included between the base and a plane parallel to the base. Hence the frustum of a wedge has six faces, two rectangles called the ends, and four trapeziums. The perpendicular distance between the ends is called the height.



(Fig. i)

FRUSTUM OF WEDGE

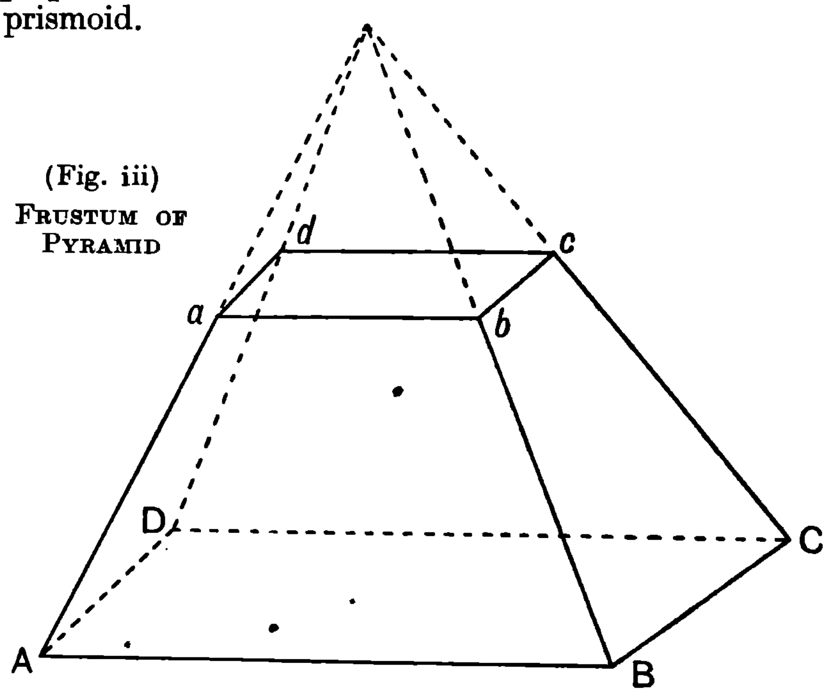
(Fig. ii)
PRISMOID



A prismoid is a solid whose ends are rectilineal figures of the same number of sides, lying in parallel planes, and the other faces are trapeziums.

The perpendicular distance between the ends is called the height of the prismoid.

(Fig. iii)
FRUSTUM OF PYRAMID



When the ends of a prismoid are similar rectilineal figures similarly situated, the prismoid is the frustum of a pyramid.

(Fig. i) shows the frustum of a wedge, (Fig. ii) a prismoid, (Fig. iii) frustum of a pyramid.

The frustum of a wedge differs from the frustum of a pyramid in that the ends are not similar and hence the edges Aa, Bb, Cc, Dd will not, if produced, meet at a point, while such edges of the frustum of a pyramid will meet as shown in (Fig. iii).

A plane drawn parallel to the base through the middle point of the height cuts any solid in the mid section. Thus any side of the mid section of a prismoid is half the sum of the corresponding sides of the ends.

The volume of the prismoid or frustum of the wedge

$$= \frac{\text{height}}{6} \text{ (sum of area of ends} + 4 \text{ times area of mid section).}$$

Examples

Ex. 1. The edge of a wedge is 18 in., the length of the base is 15 in., and the breadth of the base is 9 in., the height is 14 in. Find the volume.

$$\text{Volume} = \frac{9 \times 14}{6}(2 \times 15 + 18)$$

$$= \frac{9 \times 14 \times 48}{6} = 1008 \text{ cubic in.}$$

Ex. 2. The base of a wedge is a square, a side of which is 15 in., the edge is 24 in. and the height of the wedge is 24 in. Find the volume.

$$\text{Volume} = \frac{15 \times 24}{6}(2 \times 15 + 24)$$

$$= \frac{15 \times 24}{6} \times 54 = 3240 \text{ cubic in.}$$

Ex. 3. The section of a wedge made by a plane perpendicular to the edge is an equilateral triangle of side 8 in. Find the volume if the edge of the wedge is 18 in., and the length of the base 21 in.

Here the breadth of the base is 8 in.

$$\text{Height} = 8\frac{\sqrt{3}}{2} = 4\sqrt{3}$$

$$\therefore \text{Volume} = \frac{8 \times 4\sqrt{3}}{6} \times (2 \times 21 + 18)$$

$$= \frac{8 \times 4\sqrt{3}}{6} \times 60 = 554{\cdot}24 \text{ cubic in.}$$

Ex. 4. How many tons of earth removed in excavating a trench of which the top and bottom are rectangles ? At the top 400 ft. long, 18 ft. wide, and at the bottom 350 ft. long by 15 ft. wide. The bottom is horizontal and the depth, 12 ft. [1000 cubic ft. of earth weighs 42 tons.]

Here volume =

$$\frac{12}{6}\left[(400\times 18)+(350\times 15)+4\left(\frac{400+350}{2}\times\frac{18+15}{2}\right)\right]$$

and tonnage of earth

$$= \text{volume}\times\frac{42}{1000}$$

$$= 2\left(7200+5250+\frac{12375}{2}\right)\times\frac{42}{1000}$$

$$= 2\times\frac{37275}{2}\times\frac{42}{1000}$$

$$= 1565{\cdot}55 \text{ tons.}$$

Ex. 5. Find the volume of a prismoid, 10 ft. high, whose ends are rectangles 280 ft. by 250 ft., and 260 ft. by 190 ft.

$$\text{Volume} = \frac{10}{6}\left[(280\times 250)+(260\times 190)+4\left(\frac{280+260}{2}\times\frac{250+190}{2}\right)\right]$$

$$= \frac{10}{6}(70000+49400+237600)$$

$$= \frac{10\times 357000}{6} = 595000 \text{ cubic ft.}$$

Ex. 6. Find the cubic content of a piece of road embankment 300 ft. long, the longitudinal slope being regular, height at the ends being 5 ft. and 3 ft. respectively, the side slopes 2 (horizontal) to 1 (vertical), and the breadth at the top throughout 26 ft., the ends being vertical.

The ends are trapeziums, the mid section is also a trapezium.

$$\text{Volume} = \frac{300}{6}\left(\frac{38+26}{2}\times 3+\frac{46+26}{2}\times 5+4\times\frac{42+26}{2}\times 4\right)$$

$$= 50\times 820 = 41000 \text{ cubic ft.}$$

Exercise 13

1. Find the volume of a wedge standing upon a rectangular base measuring 7 in. and 5 in., if the height is 3 in. and the edge 4 in.

2. The edge of a wedge is 1 ft. 8 in., the area of a section of the wedge made by a plane perpendicular to the edge is 1 sq. ft. Find the volume if the length of the base is 2 ft.

3. The base of a wedge is a square of side 12 in., the height of the wedge 21 in., and the edge 27 in. Find the volume.

4. A wedge-shaped trench is 8 ft. wide and 40 yd. long at the top; the length of edge along the bottom is 32 yd. and the depth is 10 ft. How many tons of earth have been excavated? (Given 1 cubic ft. of earth weighs $92\frac{1}{2}$ lb.)

5. The triangular faces of a wedge are equally inclined to the base of which the length is 40 in., the breadth 18 in. The height and edge respectively of the wedge are 12 in. and 30 in. Find the slant surface.

6. The cross section of a wedge is an equilateral triangle on a side of 20 in. and the triangular faces are inclined to the base at an angle of 60°. If the base be 40 in. long, find the volume and slant surface.

7. Find the volume of a prismoid whose parallel ends are rectangles, measuring 16 in. by 10 in., and 9 in. by 6 in., the thickness being 4 in.

8. An excavation 120 yd. long is uniformly 42 ft. wide at the bottom and 16 ft. deep at one end, 20 ft. deep at the other end, the slope being gradual. The upper width at the deeper end is 86 ft., and at the other 74 ft. Find the cost of excavation at the rate of Rs. 5 per 1000 cubic ft.

9. The top and bottom of a reservoir are rectangles, the top is 150 ft. long by 80 ft. broad, and the bottom 120 ft. long by 70 ft. broad. If the depth is 24 ft., find the volume of water, in gallons, the reservoir contains when full to the brim. (1 cubic ft. = $6\frac{1}{4}$ gallons.)

10. A prismoid stands on a square base, and one pair of opposite slant faces are inclined to the base at an angle of 60°, the other pair at 30°. The side of the base is 16 ft. and the height $2\sqrt{3}$ ft. Find the dimensions of the top, the volume and the slant surface.

Examination Questions

11. Find the volume of a wedge, the length and breadth of the base being 5 ft. 4 in., and 9 in. respectively, the length of the edge being 3 ft. 6 in., and the height 2 ft. 4 in.

12. The edge of a wedge is 15 in., the length of the base is 24 in., and the breadth 7 in., the height is 22 in. The wedge is divided into a pyramid and a prism by a plane through one end of the edge parallel to the triangular face at the other end. Find the volume of each part.

13. A cylindrical vessel 1 ft. high, and the radius of whose base is 6 in., is full of water. A wedge whose edge is 7 in., whose base is 5 in. long and 4 in. broad, and whose height is 6 in., is gently dipped into the water so that the water runs over, it is then withdrawn. At what height in the vessel will the water now stand?

14. The edge of a wedge is 21 in., the length of the base is 27 in., the area of a section of the wedge made by a plane perpendicular to the edge is 160 sq. in. Find the volume.

15. The edge of a wedge is 21 in., the length of the base is 15 in., and the breadth 9 in., the height of the wedge is 6 in. The wedge is divided into three parts of equal heights by planes parallel to the base. Find the volume of each part.

16. The edge of a wedge is 9 ft., the length of the base is 6 ft., and the breadth is 4 ft., the height of the wedge is $2\frac{1}{2}$ ft. Find the volume.

17. The edge of a wedge is 25 in., the length of the base is 22 in., a section of the wedge made by a plane perpendicular to the edge is an equilateral triangle, each side of which is 10 in. Find the volume.

18. AE, BF, CG, DH, are the vertical edges of a cubic foot of wood whose horizontal faces are ABCD, EFGH. In AB a point M is taken 7 in. from A, and in AD a point N 5 in. from A. A portion of the cube is cut away by a plane through MFG, and then a second portion by a plane through NHG. Find the volume of the three portions into which the cube is thus divided.

19. A reservoir with slanting sides, whose base is 50 ft. by 40 ft., and top 75 ft. by 60 ft., is 15 ft. in perpendicular height. Find the number of gallons it will hold.

20. The top and bottom of a reservoir in the shape of a prismoid are rectangles, the dimensions of the top being 200 ft. by 150 ft.,

and of the bottom 160 ft. by 130 ft., its uniform depth is 12 ft. Find the cost of excavation at 1*s*. 6*d*. per cubic yard.

21. The length and breadth of a reservoir in the shape of a prismoid are 140 ft. and 80 ft. respectively; the length and the breadth of the bottom are 100 ft. and 60 ft. respectively, and the depth is 12 ft. How many cubic feet of earth were dug out?

22. A piece of timber is 1 ft. 2 in. broad and 10 in. thick at one end, and 1 ft. 6 in. broad and 1 ft. thick at the other end, and 14 ft. long. Find its volume.

23. A watercourse, 5 ft. wide at the bottom, 3 ft. deep at the upper end and having a fall of 1 ft. in 320 yd. is to be cut in a straight line on level ground. If the sides are to slope 1 in 1, find the number of cubic yards of earth to be excavated in the first mile.

24. Find the quantity of earth excavated from a railway cutting made through ground which before disturbance was a uniform inclined plane running in the same direction as the rails, the length of the cutting being 100 yd., the breadth at the bottom 12 yd., the breadth at the top at one end being 45 yd. and at the other 25 yd., and the depths of these ends being 15 yd. and 7 yd. respectively.

25. The top widths of a railway cutting are 120 and 90 ft.; their respective depths 30 ft. and 20 ft., the bottom width 30 ft., and the length of the cutting 66 yd. Find the contents in cubic yards.

26. A haystack $11\frac{1}{2}$ ft. high has an oblong base 20 ft. long and 8 ft. broad, the sides of the rectangular horizontal section 9 ft. from the ground through the eaves are 22 ft. and 8·8 ft. and the part above the eaves forms a triangular prism 22 ft. long. If 10 cubic ft. of hay weigh 1 cwt. how many tons does the whole stack weigh?

27. Find the volumes of the wedge and prismoid into which a frustum of a pyramid is cut by a plane passing through one end of its base and cutting off a portion of the top 15 in. distant from its corresponding end, the length and breadth of the base being 45 in. and 30 in respectively, those at the top being 36 in. and 24 in. respectively, and the height 40 in.

28. An excavation 858 ft. long is uniformly 50 ft. wide at the bottom; it is 18 ft. deep at one end, and gradually increases to 20 ft. deep at the other and the upper width of these ends are respectively 104 and 110 ft. Find the number of cubic yards in the excavation.

29. How many cubic feet of earth will be required to make a level embankment 1500 ft. long, 17 and 12 ft. deep at the ends, 20 ft. wide on top, with side slopes of $1\frac{1}{2}$ to 1 ?

30. State and explain the prismoidal formula. A tank is 436 ft. by 325 ft. at the top, 376 ft. by 285 ft. at the bottom, and 10 ft. deep. How many cubic feet of water will be required to fill it three quarters full if there is in the middle a circular tower of 27 ft. diameter?

31. The ends of a prismoid are rectangles, the corresponding dimensions of which are 12 ft. by 10 ft. and 8 ft. by 6 ft., the height of the prismoid is 4 ft. The prismoid is divided into two parts by a plane parallel to the ends and midway between them. Find the volume of each part.

32. A road is constructed along the greatest slope of a plane country. Top width of the road is 20 ft., side slopes are 2 horizontal to 1 vertical. Find the quantity of earth work of a portion of the road 4 chains in length. Heights of embankment at the beginning and the end of that portion of the road are respectively 10 ft. and 20 ft.

33. Find the capacity of a coal wagon the top of which measures 6 ft. 9 in. in length by 4 ft. 6 in. in breadth, the bottom 3 ft. 6 in. by 2 ft. 6 in. and the depth 4 ft.

34. A railway embankment is half a mile long, and has a uniform width of 30 ft. at the top. At one end it is 25 ft. high, and gradually decreases to the other end to 12 ft. high; the widths at the base at the ends are 120 ft. and 80 ft. respectively. Find the cost of making the embankment at Rs. 5 per 1000 cubic ft.

35. A prismoid has one end in the form of an equilateral triangle of side 2 ft., the other end in the form of a regular hexagon of side 1 ft., three sides of the hexagon being parallel to the three sides of the other end; the height is 3 ft. Find its volume.

36. Find the volume of a coal wagon, the depth of which is 47 in., the top and bottom are rectangles, the corresponding dimensions of which are 81 in. by 54 in. and 42 in. by 30 in.

37. Find the capacity of a trough in the form of a prismoid, its bottom being 48 in. long and 40 in. broad, its top 5 ft. long and 4 ft. broad, and the depth 3 ft.

38. An embankment is made upon a slope of 1 in 10; the top of the embankment is horizontal, and its section is everywhere a trapezoid. The greatest height above the slope is 57 ft., the breadth of the top 26 ft. and the slopes of the sides 1 in 1. Find the number of cubic yards in a length of 160 yd. of the embankment.

Chapter XVII

THE SPHERE

68. A sphere is a solid generated by revolving a semicircle about its diameter. Thus all straight lines drawn from the middle point of the diameter of the semicircle to the bounding surface of the sphere are equal to one another. This point is called the centre of the sphere and these straight lines, the radii.

The section of a sphere by any plane is a circle. A plane section through the centre is called a great circle.

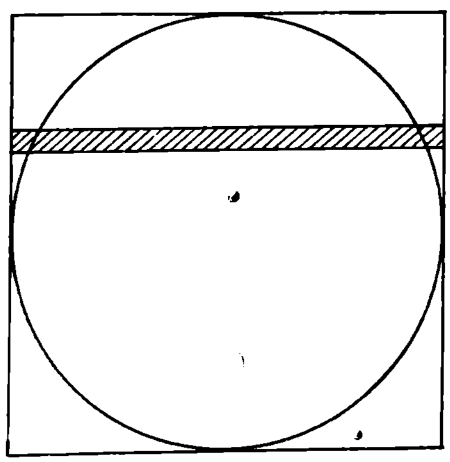
A tennis or a cricket ball is a familiar example of a sphere.

If a cylindrical tube contain a sphere which fits it exactly, any two planes perpendicular to the axis cut off belts of equal surface both from the sphere and the interior of the tube.

Thus the surface of a sphere is equal to the inner surface of the circumscribed cylinder.*

$\therefore$ The surface of the sphere

$$= \pi(2 \text{ radius of sphere}) \times 2 \text{ radius of sphere}$$
$$= 4\pi \text{ radius}^2.$$



The surface of a sphere may be divided into an infinite number of infinitely small polygons. By connecting the corners of these polygons with the centre of the sphere there are formed an infinite

* This relation was discovered by Archimedes.

number of pyramids, whose vertices are the centre and height is the radius of the sphere.

The volume of the sphere is, therefore, equal to the sum of the pyramids,

$$= \text{sum of areas of the bases of pyramids} \times \frac{r}{3}$$

But sum of area of the bases of pyramids is the surface area of the sphere.

$$\therefore \text{ the volume of sphere} = \text{surface area} \times \frac{r}{3}$$

$$= \frac{4}{3}\pi r^3.$$

Examples

Ex. 1. Find the surface and volume of a sphere whose radius is 1 ft. 9 in.

$$1 \text{ ft. } 9 \text{ in.} = 1\tfrac{3}{4} \text{ ft.} = \frac{7}{4} \text{ ft.}$$

$$\text{Surface} = 4 \times \frac{22}{7} \times \frac{21}{12} \times \frac{21}{12} = \frac{77}{2} = 38\tfrac{1}{2} \text{ sq. ft.}$$

$$\text{Volume} = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} = \frac{539}{24} = 22\tfrac{11}{24} \text{ cubic ft.}$$

Ex. 2. The diameter of a hemispherical bowl is 9 in. Find how many pints of water it holds. (1 cubic ft. = $6\frac{1}{4}$ gallons.)

$$9 \text{ in.} = \tfrac{3}{4} \text{ ft. and } r = \frac{3}{8} \text{ ft.}$$

$$\text{Volume of bowl} = \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \text{ cubic ft.}$$

$$\text{Water in pints} = \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{25}{4} \times 8 = \frac{2475}{448}$$

$$= 5 \cdot 52 \text{ pints.}$$

Ex. 3. An n-bore shot-gun is one which is such that n spherical bullets of lead of the same diameter as the barrel weigh 1 lb. If a cubic inch of lead weighs 0·41 lb., find the diameter of a 12-bore gun.

1 lb. lead contains $\frac{1}{0\cdot41}$ cubic in.

$\therefore$ Volume of bullet of a 12-bore gun $= \frac{1}{0\cdot41 \times 12}$ cubic in.

and r of bullet $= \sqrt[3]{\frac{1 \times 3 \times 7}{0\cdot41 \times 12 \times 4 \times 22}} = \sqrt[3]{\cdot048503}$

or r = ·36469, and diameter = ·72938 in.

Exercise 14

1. Find the surface and volume of the following spheres:
   (i) Radius 1 ft. 2 in.
   (ii) Diameter 2 yd. 1 ft.
   (iii) Radius 4·2 in.

2. What is the radius of a sphere whose surface is 2464 sq. ft.?

3. Find the cost of gilding a hemispherical dome 14 ft. in diameter, at the rate of 1*s.* 6*d.* per sq. yd.

4. The volume of a sphere is equal to that of a right circular cone, the radius of whose base is 4 ft., and height 3·9 ft. Find the surface of each.

5. The surface of a spherical shot is 36π sq in. Find its radius and volume.

6. You are given a set of 20 equal steel bearing balls and are asked to find the diameter of one of them as accurately as possible without actual measurement. You find that their total weight is ·934 lb., and you know that a cubic inch of steel weighs ·288 lb. Complete the calculation.

7. A hollow cylinder is closed at the ends by hemispheres. If the length of the cylinder is 8 ft., and its diameter 6 ft., find the whole external surface in sq. ft.

8. A solid figure made of cast iron (s.g. 7·2) consists of a cylinder with one plane end and one hemispherical end. Total length = 15 in., diameter = 4·2 in. Find the weight. (1 cubic ft. of water = 62·5 lb.)

9. A golf ball has an external diameter of 1·6 in. It consists of a core of diameter 0·8 in., and an outer cover. If the s.g. of the core is 1·08 and of the outer cover is 0·92, find the weight of the ball in ounces. (1 cubic ft. of water weighs 1000 oz.)

10. Find the radius of a sphere whose surface is equal to the surface of a cylinder of height 16 in., and diameter 4 in.

11. How many spherical bullets, each 1 in. in diameter, could be moulded from a rectangular block of lead 11 in. long, 8 in. wide and 5 in. thick ?

12. Find the radii of the spheres whose volumes are:

(i) $113\frac{1}{7}$ cubic in.
(ii) $179\frac{2}{3}$ cubic ft.

13. Find the volumes of the spheres whose surfaces are:

(i) 616 sq. in.
(ii) $38\frac{1}{2}$ sq. ft.

14. Find the surfaces of the spheres whose volumes are :

(i) $179\frac{2}{3}$ cubic in.
(ii) $4\frac{4}{21}$ cubic ft.

15. Find the weight (to the nearest ounce) of a spherical shell of copper, having an external diameter of one foot, and a thickness of one inch. (1 cubic inch of copper weighs 5·1 oz. nearly.)

16. A metal sphere of diameter 1 ft. is dropped into a cylindrical well, which is partly filled with water. The diameter of the well is 4 ft. If the sphere is completely submerged, by how much will the surface of the water be raised ?

17. The external diameter of a hemispherical bowl is 1 ft. 10 in. Find its thickness, if its whole surface is 1454·5608 sq. in.

18. The greatest possible sphere is turned from a cubical block of deal. If the weight of the wood removed is 238·857 lb., find the diameter of the sphere. (1 cubic ft. of deal weighs 57 lb.)

19. Find to the nearest tenth of an inch, the internal diameter of a hemispherical vessel capable of containing 11 gallons. (1 gallon = 277·25 cubic in.)

20. A spherical iron shell is 32 in. in external diameter. Find the thickness if the shell is capable of containing 50 gallons of liquid. (1 cubic ft. = $6\frac{1}{4}$ gallons.)

Examination Questions

21. A solid consisting of a right cone standing on a hemisphere is placed in a right cylinder full of water, and touches the bottom. Find the volume of water displaced, having given that the radius of the cylinder is 3 ft., and its height 4 ft., the radius of the hemisphere 2 ft., and the height of the cone 4 ft.

22. A spherical cannon ball, 9 in. in diameter is melted and cast into a conical mould, the base of which is 18 in. in diameter. Find the height of the cone.

23. It is calculated that the heat received by the earth from the sun in a year would be sufficient to melt a layer of ice 100 ft. thick all over the surface of the earth. Assuming the earth to be a sphere of radius 4000 miles, find the volume of this ice in cubic miles.

24. A solid iron cube, the edge of which is 2 ft. in length, and a solid iron sphere the radius of which is 1 ft., are thrown into a cubical tank which is 6 ft. across, and is half filled with water. Find the rise of the surface of the water in inches to five places of decimals if they both be completely immersed. ($\pi = 3{\cdot}14159$.)

25. Find the weight of a pyramid of iron such that its height is 8 in. and its base is an equilateral triangle, each side being 2 in., supposing a ball of iron 4 in. in diameter to weigh 9 lb.

26. A hemispherical basin 15 ft. in diameter will hold one hundred and twenty times as much as a cylindrical tub, the depth of which is 1 ft. 6 in. Find the diameter of the tub.

27. A solid ball, 4 in. in radius, of a certain material weighs 8 lb. Find the weight of a spherical shell of that material, the internal diameter of which is 8 in., and the external diameter 10 in.

28. A solid is composed of a cone and hemisphere on opposite sides of the circular base, the diameter of which is 2 ft. and the vertical angle of the cone is a right angle. Find the volume of the whole.

29. A hemispherical punch bowl is 5 ft. 6 in. round the brim. Supposing it to be half full, how many persons may be served from it in hemispherical glasses $1\frac{3}{4}$ in. in diameter at the top ?

30. A 2 ft. tube is partly filled with water, a sphere that exactly fits the tube is placed in it, and the water is found to rise just to the highest point of the sphere. How much water was there in the tube ?

31. A cast iron shell has an external diameter of 1 ft. The metal is 2 in. thick. Find the weight of the shell. (1 cubic ft. of iron weighs 450 lb.)

32. A hemisphere of lead of radius 6 in. is cast into a solid cube. Find to three decimal places the length of an edge of the cube. ($\pi = 3{\cdot}14159$.)

33. Find the thickness of a shell whose outer diameter measures 7 in. if it weighs half as much as a solid ball of the same diameter.

34. A hemisphere and a right cone lie on opposite sides of a common base of 4 ft. diameter, and the cone is right angled at the vertex. If a cylinder should circumscribe them in this position, how much additional space is thereby enclosed?

35. If 30 cubic in. of gunpowder weigh 1 lb., find the diameter of a hollow sphere which will hold 11 lb.

36. A heavy iron cylinder with hemispherical ends is immersed in water; find the amount of water displaced, the solid's extreme length being 12 ft. and diameter 3 ft.

37. What is the weight of an iron shell, the external and internal diameters of which are 13 in. and 10 in. respectively, if an iron ball of 4 in. diameter weighs 9 lb.?

38. A hemispherical bowl whose internal radius is 1 ft. is filled with water and kept so that the rim is horizontal. A cone whose vertical angle is 90° is placed with its axis vertical, its base at the level of the rim of the bowl, and its apex at the centre of the bottom of the bowl. Find the quantity of water left in the bowl after the intrusion of the cone.

39. Determine the weight of a spherical cast-iron shell whose inner and outer diameters are $6\frac{1}{2}$ in. and $7\frac{1}{2}$ in. respectively, the weight of a cubic foot of iron being 450 lb.

40. Find the expense of painting a cylindrical pontoon with hemispherical ends at 6*d.* per square yard, the length of the cylindrical part being 19 ft. 4 in. and the common diameter of the cylinder and hemispheres being 2 ft. 8 in.

41. A circular room has perpendicular walls 15 ft. high, the diameter of the room being 28 ft.; the roof is a hemispherical dome. Find the cost of plastering the whole surface at 9*d.* per square foot.

42. Find the volume of a sphere when its surface is equal to that of a circle 9 ft. in diameter.

43. A wrought iron cylindrical boiler 10 ft. long, 4 ft. in diameter, and $\frac{3}{8}$ in. thick (inside measurements), is closed by hemispherical ends. Find the external surface.

44. A cylinder 12 ft. high and 6 ft. in diameter is surmounted by a cone also 6 ft. in diameter and 4 ft. high. Find the radius of a hemisphere whose entire surface is equal to the united curved surfaces of the cone and the cylinder.

45. The price of a ball at 1*d.* the cubic in. is as great as the cost of gilding it at 3*d.* the sq. in. What is its diameter ?

46. A cylinder 24 ft. long and 4 ft. in diameter is closed by a hemisphere at each end. Find the area of the whole surface.

47. A sphere has the same number of cubic feet in its volume as it has square feet in its surface. Find the diameter.

48. Assuming the earth to be a sphere with a diameter of 5,000,000 ft., find the area of its surface in square miles.

49. What is the surface of a sphere whose diameter is 21 in. ?

50. A cathedral has two spires and a dome ; each of the former consists, in the upper part, of a pyramid 60 ft. high, standing on a square base, of which a side is 20 ft. The dome is a hemisphere of 40 ft. radius. Find the cost of covering the three with lead at $7\frac{1}{2}$*d.* per sq. foot. ($\pi = 3{\cdot}1416$.)

CHAPTER XVIII

CONSTRUCTION OF SCALES

69. It is not convenient to make a full-size plan of a table, a box, a building, or any field on paper. The drawing or map is therefore, made to scale, that is, each line in the plan is drawn with a fixed proportion.

Suppose, for example, that in a drawing, a table 3 ft. by 2 ft., is drawn 3 in. by 2 in. Then the drawing is made to a scale of 1 foot to an inch. Thus the length of a line on the drawing is 1/12th part of the actual length it represents. This fraction, which represents the proportion of the drawing to the object, is called the "Representative fraction," or "R.F." of the drawing. The unit should be the numerator of the R.F.

If a plan is drawn to a scale of 10 ft. to 1 in., a length of 5 ft. 6 in. will be drawn 11/20th of one inch, a length of 7 ft. 4 in., 11/15th, a length of 3 ft. 2 in., 19/60th and so on. But to mathematically calculate each representative length of the plan is a tedious procedure and is not resorted to. For obtaining readily the representative lengths a graduated straight line, called the scale, is used.

The examples below explain the method of constructing the scales.

Ex. 1. Construct a scale of 10 feet to an inch.

Draw a line 6 in. long which will represent 60 ft. (A scale is usually drawn about 6 in. long.) Divide the line into 6 equal parts. Divide the left hand division into 10 equal parts each of such parts will represent 1 foot. To complete the scale, ink in two more parallel lines about 1/16th inch apart above the divided line. Draw perpendiculars from the divisions, those from the primary divisions should be drawn up to the top line, while those from the secondary divisions, up to the middle line. The right hand point of the left most primary division is invariably marked 0, and starting from that point primary divisions are numbered from left to right and secondary divisions from right to left.

Figure on the following page is a specimen of the above scale. To take a length of 37 ft. from this scale place one point of the divider at the division numbered 30, and spread the other point up to the subdivision numbered 7.

Ex. 2. Construct a scale of 11 feet to an inch.

To construct this scale take a line $6\frac{4}{11}$ in. long*, which will represent 70 ft. Divide this line into 7 parts each part representing 10 ft., and divide the left most division into 10 parts. Number the scale and complete as the specimen overleaf :

In this scale if the line is divided into one inch lengths and the divisions are numbered 11, 22, and 33, etc., the purposes of the scale will not serve well. For, in that case, the measurements cannot be taken readily. A scale is usually divided to units, tens, or hundreds, etc.

Ex. 3. Construct a scale of 16 inches to 1 mile, showing chains and links.†

Here

$$\begin{aligned} 1 \text{ mile or } 80 \text{ chains} &= 16 \text{ in.} \\ 25 \text{ chains} &= 5 \text{ in.} \end{aligned}$$

Take a line 5 in. long, divide it into 25 parts. Divide the left most division into 5 parts. Thus a primary division represents 1 chain, and a secondary division 20 links. Number and complete the scale as the specimen overleaf.

R.F. of this scale is $\frac{1}{5\times66\times12} = \frac{1}{3960}$

70. The above scales are called simple scales. It will be seen from the above examples that in a simple scale, the magnitude of the smallest measure readable, is limited to about 1/25th of an inch, as in the Ex. 3. For obtaining smaller measures the method of drawing diagonal scale is adopted.

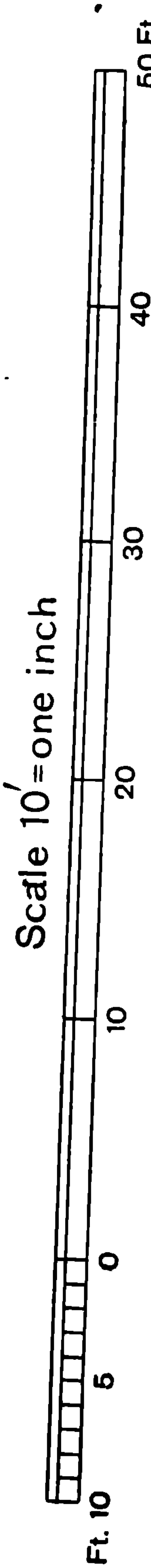
The examples below will explain the method.

Ex. 4. Draw a scale, 10 feet to 1 inch to measure feet and inches.

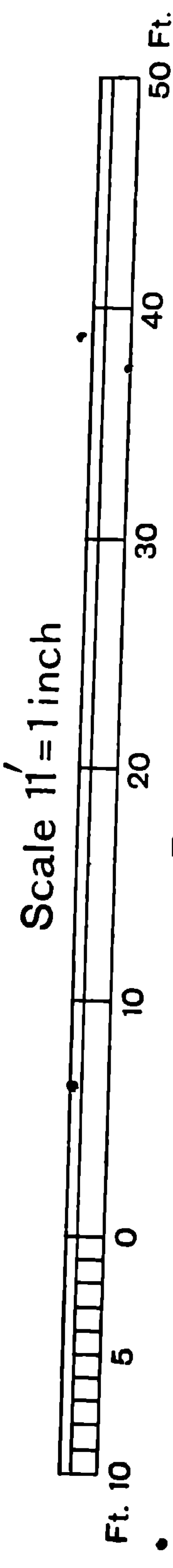
In the Ex. 1 this scale has been drawn to measure up to 1 foot only, and it is not possible to divide each foot-division into

* To get this fractional length, first take a line 7 in. long then divide the last inch into 11 parts of which take 4 parts.

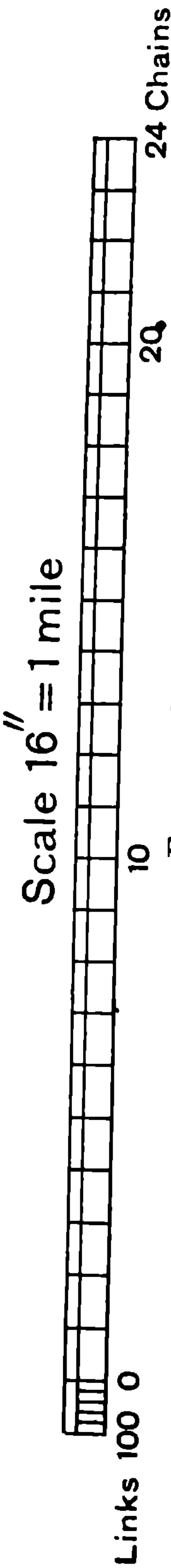
† The village maps prepared in connection with the settlement operations are drawn to this scale.



Example 1



Example 2



Example 3

12 parts to measure inches. We therefore follow the diagonal method as here explained.

Take a line 6 in. long and divide it in the same way as in Ex. 1. At the extreme left of the line erect a perpendicular about an inch or so long; on this perpendicular lay off 12 equidistant points, and through them draw 10 lines parallel to the scale line. Draw perpendiculars upon the scale line from the primary divisions to meet the top line which must have now been divided into 6 equal parts by the perpendiculars. Divide the left most division of the top line into 10 parts, thus the top line is divided exactly in the same way as the bottom scale line. Next draw the diagonals as shown in the specimen. The first diagonal is drawn from the zero in the bottom line to the first left hand sub-division of the top line. Number the scale as shown.

In this scale the line ab measures 37 ft. 5 in.

Ex. 5. Construct a diagonal scale of 16 inches to 1 mile.

Take a line 5 in. long and divide it into 5 primary divisions and subdivide the left division to 5 parts to measure a single chain. Draw a perpendicular as in Ex. 4 above and take 10 equidistant points in it. Draw parallels and perpendiculars. Divide the left hand division of the top line into 5 parts and draw diagonals as shown in the specimen.

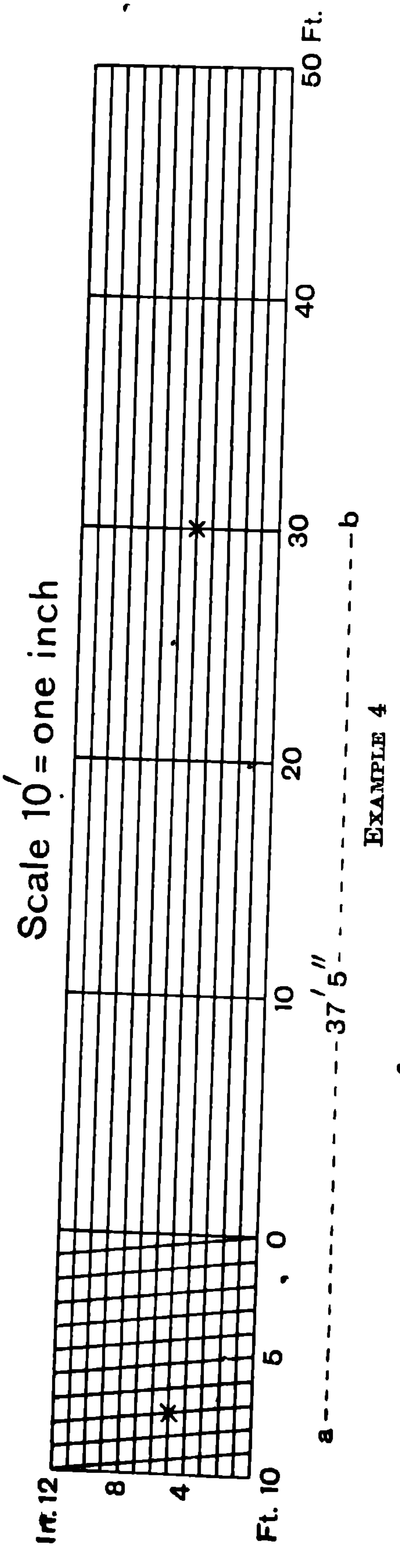
In the above scale the line ab measures 17 chains 40 links.

To measure 17 chains 45 links draw a parallel midway between the line measuring 17 chains 40 links and 17 chains 50 links.

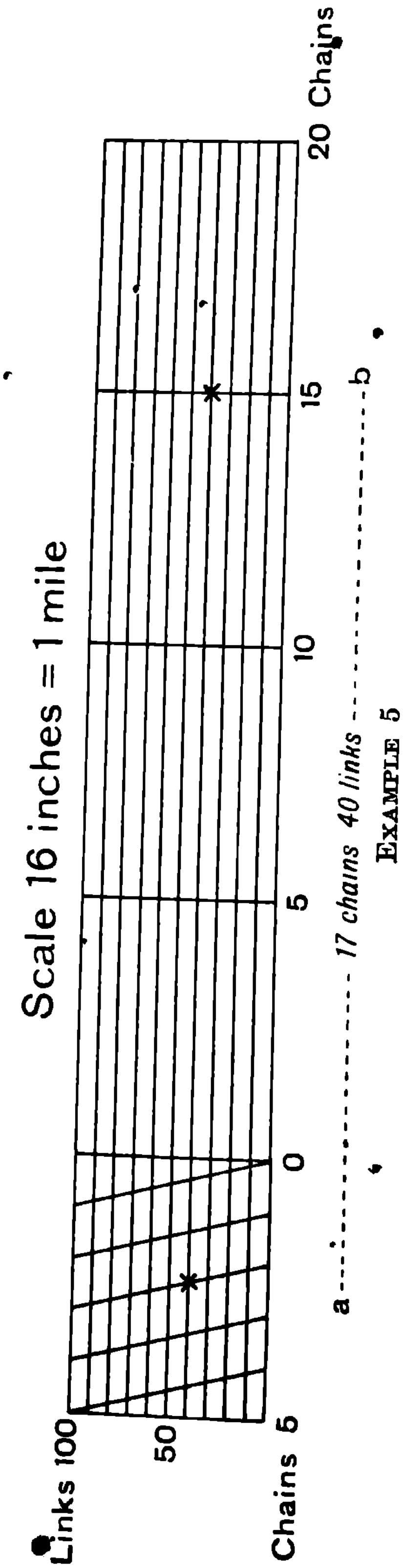
In practice, however, such a parallel is not actually drawn on the scale but the measurement is taken on such an imaginary line.

Exercise 15

1. Construct a simple scale of 8 feet to 1 inch.
2. Construct a simple scale of 50 miles to 1 inch.
3. Construct a simple scale of 100 feet to 1 inch.
4. Construct a diagonal scale of 4 yards to 1 inch to measure yards, feet and inches.
5. Construct a diagonal scale of 8 inches to a mile, to measure chains and links.



EXAMPLE 4



EXAMPLE 5

Chapter XIX

ELEMENTARY SURVEYING

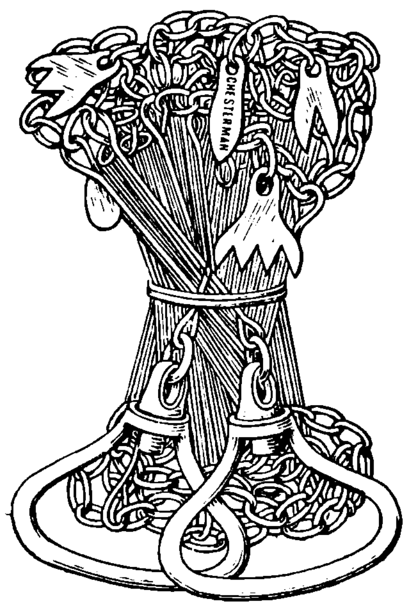
71. Land surveying* is that branch of the applied science which treats of the measurement of an area with a view to prepare a map or plan of the area and to compute the quantity of land the area contains.

Mathematically the process is classified as (i) triangulation and (ii) traversing. It is beyond the scope of this very elementary work to deal with except the simple form of the former.

72. The chain is used for measuring the length of any straight line on the ground, required in a survey. The chains are made of strong iron, or steel wire, and are of different lengths, but 100 feet and 66 feet long chains are in common use. The 66 feet chain is called the Gunter's chain, so named after the inventor, Dr. Edmund Gunter.

Each of the above chains is divided into 100 links, connected with each other by means of small rings. This arrangement allows the chain being folded to a short length.

Figure in the margin shows a folded chain.



* Land surveying is the oldest of the applied sciences.—J. Park.
The simple beginnings of the art of surveying and mapping appear to have originated in Egypt or Babylonia more than 5000 years ago.

The figure below shows a chain (in part only) stretched on the ground.

Obviously a link in a 100-foot chain is 1 foot long and that in a Gunter's chain is ·66 feet or 7·92 inches.

The 100-foot chain is used for surveys in which the area and distances are required in square feet and feet as in the case of laying out a road, a tank, a building, etc. In the case of the survey of a village, a tenure or a holding in which the area is required in acres and decimals of acres the use of Gunter's chain is of advantage as will be clear from the following:

1 acre = 4840 sq. yd. = 43560 sq. ft.

1 sq. chain = 66 ft × 66 ft. = 4356 sq. ft.

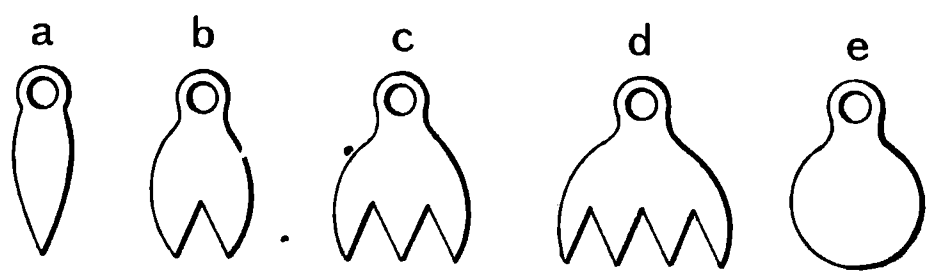
∴ 1 acre = 10 sq. chains, and

·1 acre = 1 sq. chain

·01 acre = ·1 sq. chain.

If, therefore, the unit of measure be Gunter's chain and links, the area is obtained in sq. chains and decimals, and by removing the decimal point one figure to the left, the result is obtained in acres.

At every tenth link from each end of the chain a piece of brass or other metal with notches or points, as shown below, is fixed to denote the number of units of ten links.



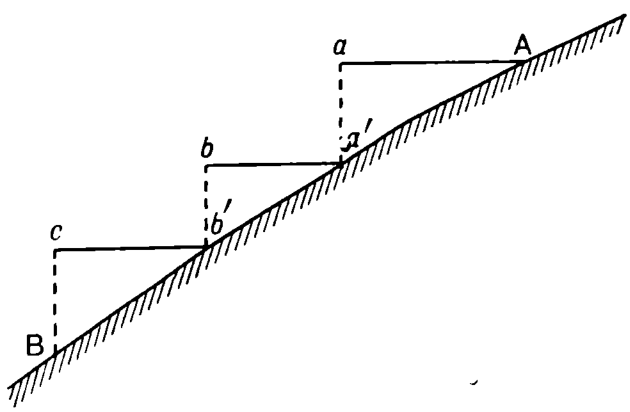
The notches vary in number according to their positions from the nearest end of the chain. A round plain piece denotes the middle point of the chain. This arrangement of dividing the chain has the advantage of its being used from either end. The brass swivel handle at each end forms part of the chain.

Besides the chains described above, there is a 30-ft. chain used for land measurement in Bengal. This chain is divided into kathas and chataks. But this kind of chain is becoming obsolete now.

Iron or steel arrows about 15 in. long are used with the chain, one being pitched at the end of each chain stretched on the ground.

For measuring a line on the ground a flag is planted at one end of the line and measurement is started from the other end. The front chain-man drags the chain, being guided by the back chain-man for a correct alignment. Generally, the front chain-man begins the measurement of a line with ten arrows in his hand and the arrows pitched by him at the end of each chain are collected by the back man. A change of arrows takes place when all the ten are in the hand of the backman, such a change indicating that ten chains have been measured. Some people prefer using eleven arrows, which allows of one being left on the ground when the back chain-man can hand over ten to the front chain-man.

While measuring with the chain it should be borne in mind that the horizontal* measurement is what is required. When, therefore, the ground is out of level, the chain should be stretched horizontally. In measuring along steep slopes the chain may be cut to short lengths, and horizontal measurements obtained by stepping. Figure below explains this method.



* A plan is defined in the *Imperial Dictionary* to be "properly the representative of anything drawn; a plane, as a map or chart. The term plan may be applied to the draught or representation of any projected work on paper, or on a plane surface, as the plan of a town or city." In other words a plan is a horizontal representation of the features of the ground. All measurements must, therefore, be reduced to a horizontal plane.

Horizontal distance AB = Aa+a′b+b′c.

The accuracy of all survey works depends, to a very great extent, on the correct measurement of distances on the ground. The chain should always be of the true length.

To ensure this end it should be tested against a standard* every day. If found incorrect the length can be adjusted by adding, or removing some small rings which join the links. While correcting the chain care should be taken that the centre index remains midway between the ends.

In the absence of the standard, the chain can be tested by standard steel bars, steel tape, or even by a wooden pole or rod.

For long chain lines, especially over undulating ground, it is often necessary to range out, or align, intermediate points, and flags placed over such points, or else, if the flag at the end of the line be not visible from any part of the line, it will not be possible to complete the measurement.

If, for any reason whatever, a line is measured with a chain of inaccurate length, the measurement can be corrected by the following rule of proportions :

Correct length of the chain : Length of the wrong chain :: Wrong measurement of the chain : Correct measurement of the line.

As for example, a line is measured with a chain 3 in. too long to be 33 chains 66 links. What is the correct length of the line?

Here

66 ft. : 66 ft.—3 in. :: 33·66 : correct length

$$\text{Or correct length} = \frac{66\frac{1}{4} \times 33{\cdot}66}{66}$$

$$= \frac{66{\cdot}25 \times 33{\cdot}66}{66} = \frac{265 \times 33{\cdot}66}{66 \times 4} = 33{\cdot}7875 \text{ chains.}$$

Or 33 chains 78·75 links.

* At the headquarters of almost every district of Bengal there is a standard laid out by Government for checking chains, optical squares and magnetic compasses.

In the case of area the ratio will stand as under :

Square of the length of correct chain : Square of the length of wrong chain :: Area obtained : Correct area.

As, for example, supposing the area of a field is found to be 1·65 acres, by measuring it with a chain which is 3 in. too short, what is the correct area ?

Here

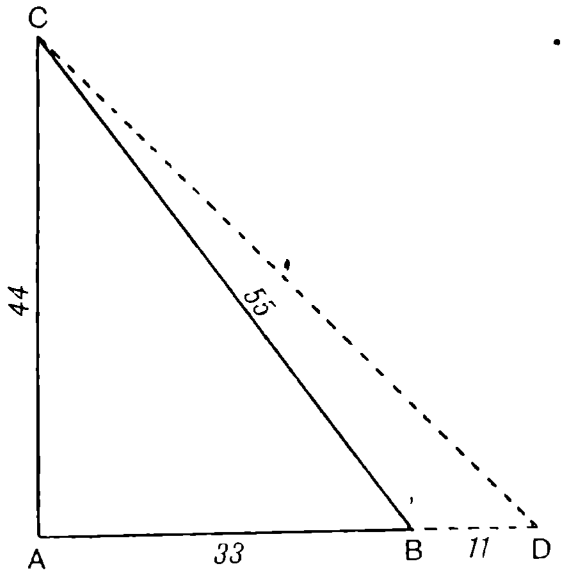
$$66^2 : (65\tfrac{3}{4})^2 :: 1{\cdot}65 : \text{correct area}$$

·025

$$\therefore \text{ Correct area} = \frac{263 \times 263 \times 1{\cdot}65}{66 \times 66 \times 4 \times 4}$$

$= 1{\cdot}638$ acres nearly.

Different known angles can be laid out on the ground with the help of a chain. (i) To lay out a right angle: measure a line AB on the ground to be 33 links. Hold one end of the chain at A and one link short from the other end of the chain at B. Keeping these two points firm stretch the chain by holding the 44th link, then the angle at A will be a right angle.



Another method : measure a line AB on the ground to be 50 links, with the middle point at C. Hold one end of the chain at A and the other end at B. Stretch the chain now holding the middle point of it. Supposing the middle point reaches D. Join DC, which will be at right angles to AB.

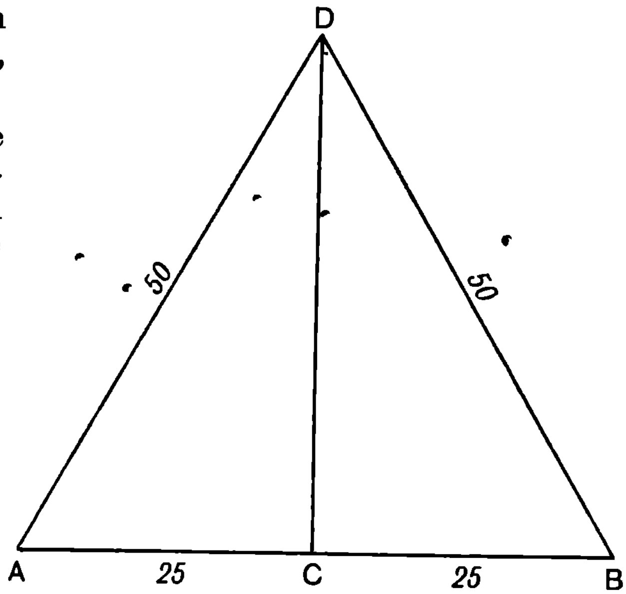
(ii) To lay out an angle of 60° :

In the above diagram each of the angles, DAB, ABD and ADB is 60°.

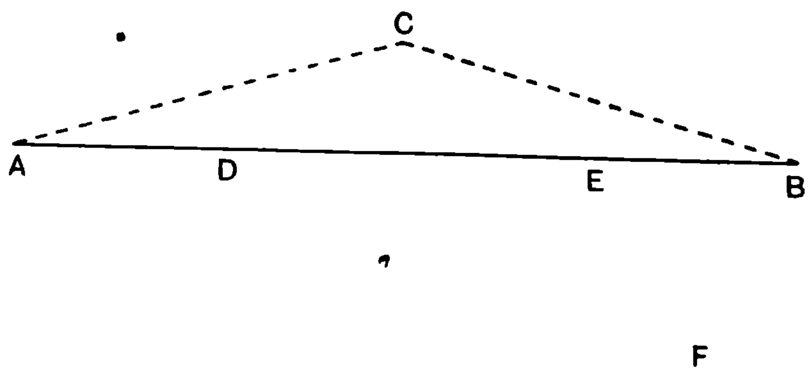
(iii) To lay out an angle of 30°. In the same diagram the angles ADC and CDB each is an angle of 30°.

(iv) To lay out an angle of 45°.

In the diagram under (i) above produce AB to D and make AD to be 44 links. Join CD. Each of the angles ACD and CDA is 45°.



73. In the diagram below A and B are two points on the ground, their relative position is determined by measuring the distance between them.



To find the relative position of C with respect to A and B the distance of C from A and from B can be measured. If it is not possible or convenient to measure from either A or B then the position of C may be measured from D and E or any other couple of points on the line AB.

Another point F may also be connected with the line AB in a similar manner. But the method is laborious and practically impossible to execute when there are a good many points, as there must be in a survey, lying on both sides of the line AB. Therefore the objects situated near the chain line within a reasonable distance of, say, one chain, are connected with the chain line by drawing perpendiculars from such objects on to the chain line. It is only in the case of an isolated object the surveyor has recourse to the other method enumerated above for connecting C or F with the line AB.

The perpendiculars mentioned above are called "offsets." For measurement of the offsets a linen tape, a light chain, or a bamboo or wooden rod 10 or 20 links long are used. For determining the point on the chain line at which the perpendicular or offset will fall the following different methods are employed.

First: By means of a linen tape for short offsets not usually exceeding about 20 ft. in length. The outer end of the linen tape is held by one man at the object from which the perpendicular is to be drawn, another man opens out the tape and determines the shortest distance on the chain line by moving up and down. [Theoretically speaking there is no objection to employ this method for long offsets, but practically when long offsets are determined by this method the result will invariably be doubtful, and hence this method is not recommended for except very short offsets.]

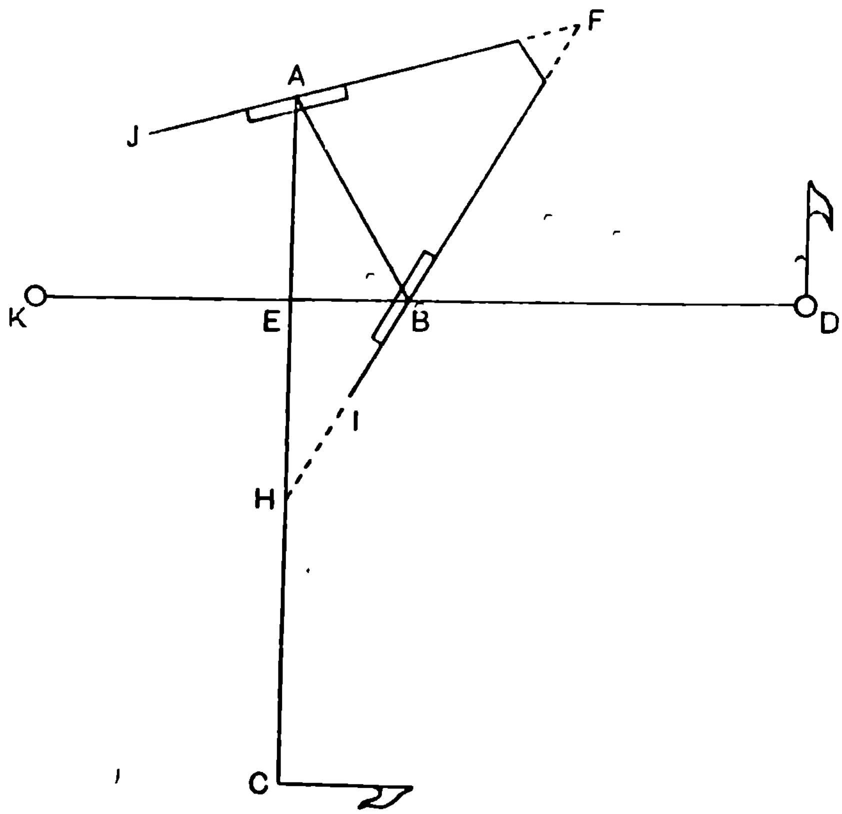
Second: By means of a cross-staff. The cross-staff is a small square board, with two lines cut at right angles at the centre of it, mounted on a 5 ft. rod, fitted with an iron spike. Placing the staff at any point on the chain line if one cut-line of the board is turned towards the flag at the end of the chain line, any object that may be lying in the direction of the other cut-line of the board will be at right angles with the chain line. From the above description it will be seen that with the cross-staff it is convenient to erect a perpendicular on a chain line from a given point in it. But in a survey it is invariably required to draw perpendicular on the chain line from a point outside it, which, with the help of a cross-staff, is a tedious task. The cross-staff is, therefore, seldom used in the field, and it has practically been ousted by the optical square. But it should be remembered that the cross-staff was one of the earliest instruments used in surveying.*

Third: By means of an optical square. The optical square is a reflecting instrument. It resembles a hollow wedge-shaped box of about 2 in. sides and 1¼ in. depth, having a handle about 3 in. long fixed below. Inside the box there are two mirrors set at an angle of 45°.

* Undoubtedly the cross-staff is the same instrument as the *diopter* of Hero of Alexandria (130 B.C.).

The construction of this instrument is based on the following principle :

In the diagram below KD is a chain line and C an object outside it.



JFI is the optical square ; angle F is 45°. A and B are two mirrors. Reflection of the flag at C first falls on the mirror A along the line CA. The same reflection, emerging from A, falls on the mirror B along the line AB. It again emerges from the mirror B along the line BK.

Angle JAC, made by the incident ray CA with the plane of the mirror A is equal to the angle FAB, made by the emergent ray AB. Again angle FBA made by the incident ray BA with the plane of the mirror B is equal to the angle HBE made by the emergent ray BE.

Thus, $\angle EBH = \angle ABF$

$$\angle FAB = \angle JAC = \angle AHF(\angle BHE) + \angle AFH(\angle BFA)$$

$$\angle EBH + \angle BHE + \angle HEB = 180° = \angle ABF + \angle BFA + \angle FAB$$

$$= \angle EBH + \angle BFA + \angle BHE + \angle BFA.$$

$$\therefore \angle HEB = 2\angle BFA.$$

If $\angle BFA$ be 45°, $\angle HEB$ will be 90°.

To use the optical square it is held in one hand by the handle with the hollow face towards the object C. The observer then moves up and down on the chain line KD as may be necessary until the reflected image of the flag at C, in the mirror B appears in one line with the flag at D, seen through the opening above the mirror B.

The position of the observer's toe on the chain line is usually taken as the point E, which is the point where the perpendicular from C falls on the chain line. To determine this point E more precisely a plummet may be suspended from the ring below the handle of the optical square. But such a precision in the measurement of an offset is superfluous.

The optical square is liable to go out of adjustment; the accuracy of it should, therefore, be tested frequently. It may be tested against a standard laid out at the headquarters of every district in Bengal, mention of which has been made before. In the absence of a standard the optical square can be tested on a chain line as KD in the last diagram. This method is very simple. First draw perpendicular on KD from C facing towards D. Let it fall at E. Then turn back and draw perpendicular again from C facing towards K, this time seeing the image of the flag at C in the same line with the flag at K. If this time also the perpendicular falls at E, then the instrument is in adjustment. If, however, the perpendicular does not fall at E at the second time and it falls either above or below E, it must be concluded that the angle of inclination of the mirrors is not 45° and the remedy lies in increasing or decreasing the angle by turning the screws by which the mirrors are set.

When using the optical square the top should be kept horizontal or else it will give inaccurate results.

74. To complete the survey of an area with the chain, the area has to be divided into triangles.* Such triangles should be as large as the nature of the ground permits and be made to approach, as nearly as possible, the form of equilateral triangles, very acute or very obtuse angles being carefully avoided.

To check† the accuracy of each triangle, "Tie" or "Check"

* The principle is enunciated in Euclid I–7 which sets forth that: "Upon the same base and on the same side of it, there can not be two triangles that have their sides which are terminated at one extremity of the base equal to one another, and likewise those which are terminated at the other extremity equal to one another." From this it will be seen that the sides being constant, the form and position of the triangle are fixed and unalterable.

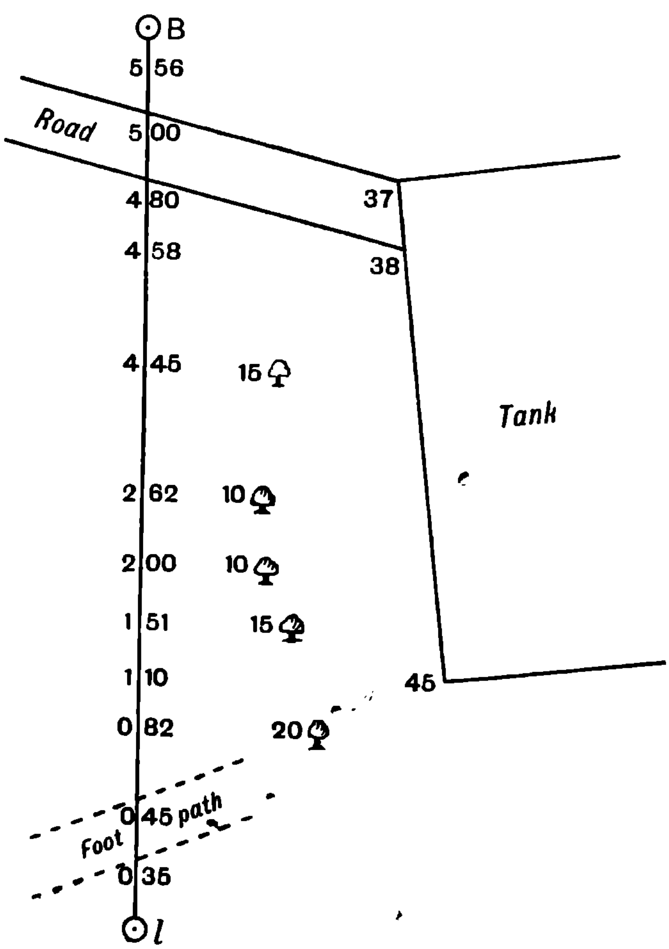
† In all survey works, whatever instruments are used, and whatever system of measurement is adopted, there is one rule imperative to all. The measurements must be checked and their accuracy proved by means of other measurements, linear or angular. Only in the case of offsets is anything to rest on a single measurement, and if great accuracy is required supplementary measurements in this case also must be taken.

lines should be run from an apex to a point about the middle of the opposite side. Tie lines may also be placed from about the middle of one side to about the middle of another within each triangle. The interior details may be filled in by taking offsets from the sides of the triangle, from the check lines, or, if necessary, from any other lines drawn inside the triangle.

(Note: In land surveying it is absolutely forbidden to run a line across another which has been previously measured.)

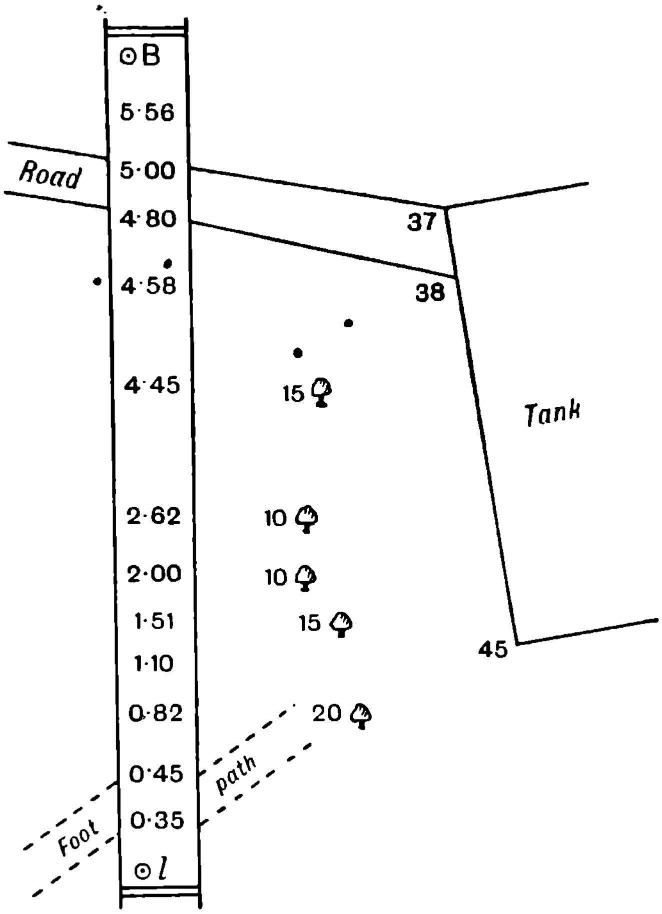
Before commencing the actual measurement in the field it is necessary to make a careful reconnaissance of the area to be surveyed, and to draw a rough hand sketch, showing the main features of the ground. The position of the triangles and of the check and other lines that will suit the case best are then shown in the sketch.

75. From the measurements obtained in the field a plan may be drawn on the spot. But it is not always convenient to do so. The measurements and other particulars are, therefore, recorded at the first instance and the plan is prepared later.

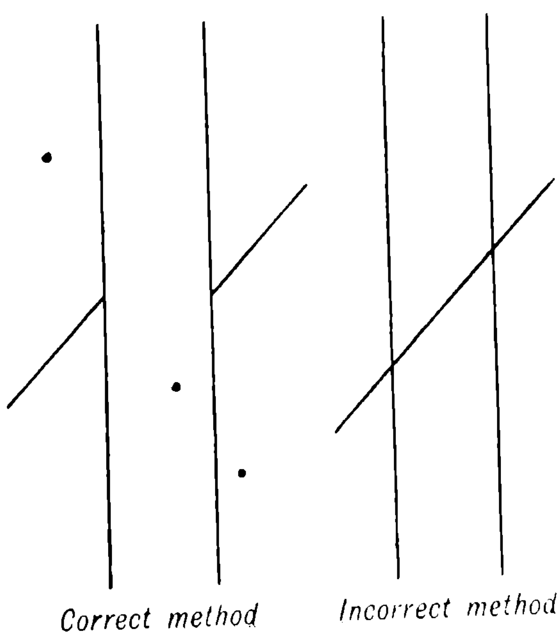


The measurements may be recorded in different ways. Firstly, they may be noted on the sketch itself, but if the measurements be too numerous, the space in the sketch may not be adequate to record them with any amount of distinctness. Notes with regard to each line may, therefore, be recorded separately at a page of a book, called the field book, in the manner shown here:

This system of writing the field book has, however, this disadvantage, that the distances along the chain line may mix up with the offset distances. To avoid this, the chain line is assumed to be an open column and the distances along the chain line are recorded within this column as shown below :



The offsets are entered on the right or left of the column, as the objects lie on the ground. When a line crosses the chain line it should meet the column in the field book at one point and leave the column at a point just opposite. A very common mistake committed by beginners in a case like this is shown on the margin :



In the field book the approximate lie of each line should be noted. The field book should be so complete that the work may be understood not only by the surveyor himself, but also by anyone who may afterwards be required independently to plot the survey who may have never seen the ground.

76. Maps are drawn at the first stage in pencil. The scale of the map is decided according to the details that should be shown. The scale varies according to the value and importance of the land surveyed, and the use which the map may be put to. It is wise to decide the scale of the map before the actual survey begins, so that the details may be measured up to the minuteness that could be distinctly shown on the map remembering that one-hundredth of an inch is not larger than a sharp pencil-point.

In drawing any plan, the first step is to draw the scale at a convenient part of the paper, so that, in the event of any shrinkage taking place, the scale will be equally affected. The main triangles are then drawn and lastly the other lines and offsets.

In plotting a plan the following points should be noted :

(i) The starting station should be so fixed that the entire plot is centrally placed on the paper. To ensure this end, the main triangles may be roughly plotted on tracing or thin paper to ascertain the shape of the plan and the space it will occupy.

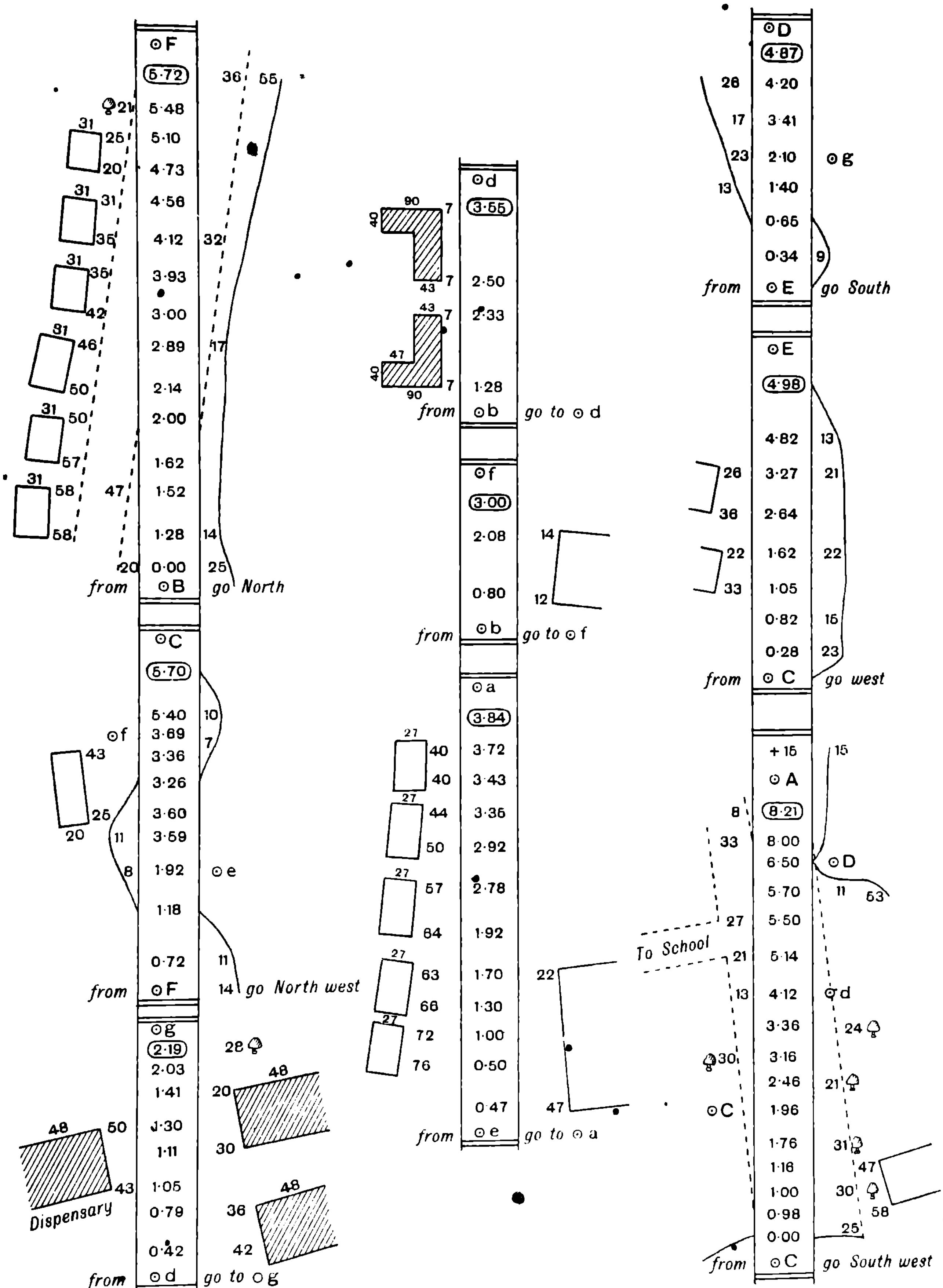
(ii) When a point is determined by the intersection of two arcs as is done in plotting the triangle, both the arcs need not be drawn, the point being marked on one arc drawn. The same observation applies to a point to be determined on one straight line by the intersection of another.

(iii) A straight line should never be obtained by the prolongation of a short line. All lines should be drawn long enough at once to obviate the necessity of prolonging them.

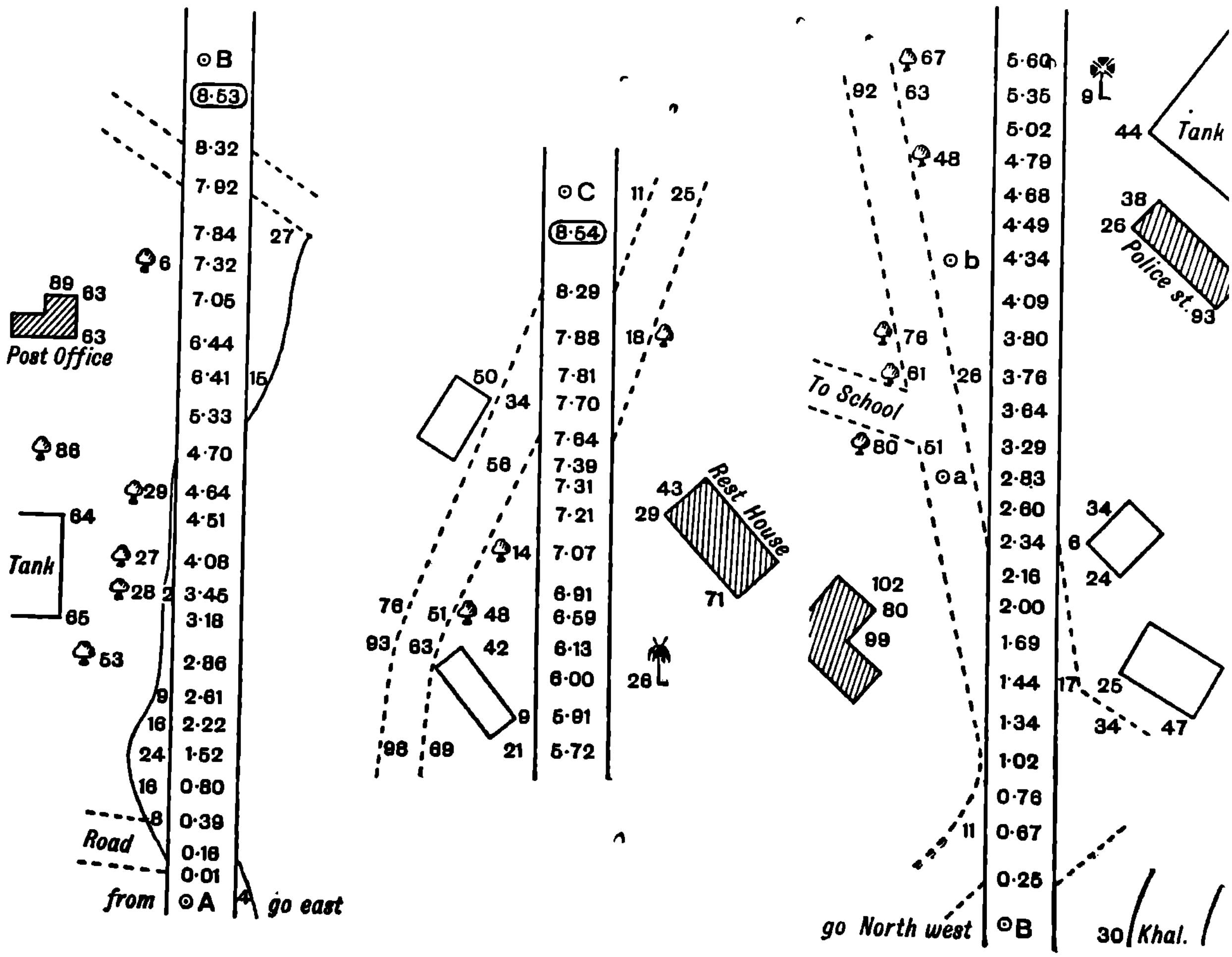
(iv) The two side lines of the paper on which the plan is drawn should point north and south. The north line of the map should be drawn. All printing and symbols of trees, etc., should head towards the north. In a chain survey map the north line is drawn approximately from the direction of the lines given in the field book.

In maps and plans there are three things to be desired : First, correctness, without which a plan is worse than useless. Secondly, clearness, in order that every detail may be understood. Thirdly, beauty of execution, which adds much to the value of a plan.

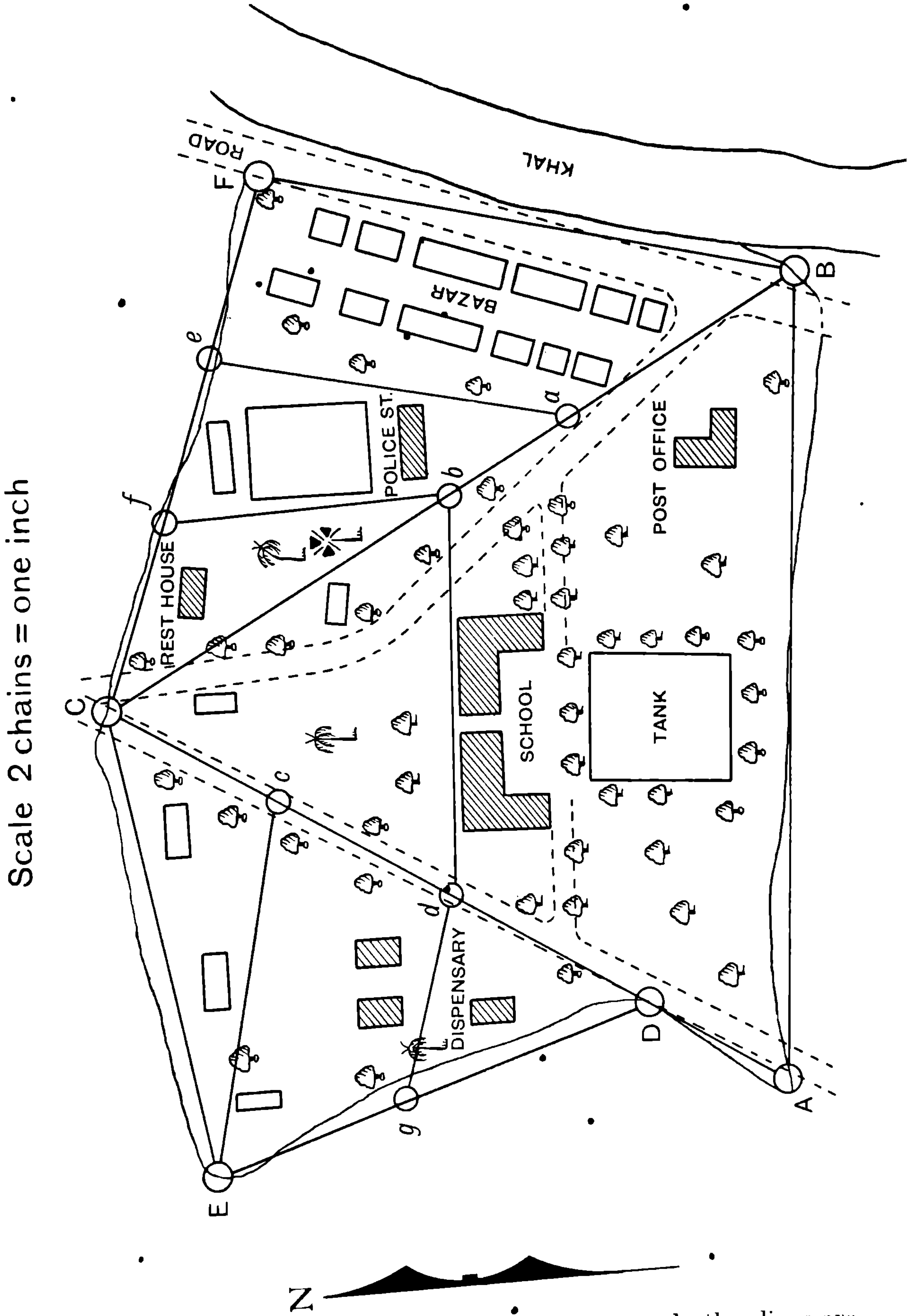
An example of the complete chain survey of an area is given on the following pages.



⊙F
5·72
5·48
5·10
4·73
4·56
4·12
3·93
3·00
2·89
2·14
2·00
1·62
1·52
1·28
0·00
from ⊙B go North
⊙C
5·70
5·40
3·69
3·36
3·26
3·60
3·59
1·92
1·18
0·72
from ⊙F go North west
⊙f
⊙e
⊙g
2·19
2·03
1·41
1·30
1·11
1·05
0·79
0·42
Dispensary
from ⊙d go to ⊙g
⊙d
3·55
2·50
2·33
1·28
from ⊙b go to ⊙d
⊙f
3·00
2·08
0·80
from ⊙b go to ⊙f
⊙a
3·84
3·72
3·43
3·36
2·92
2·78
1·92
1·70
1·30
1·00
0·50
0·47
from ⊙e go to ⊙a
⊙D
4·87
4·20
3·41
2·10
1·40
0·65
0·34
from ⊙E go South
⊙g
⊙E
4·98
4·82
3·27
2·64
1·62
1·05
0·82
0·28
from ⊙C go west
+15
⊙A
8·21
8·00
6·50
5·70
5·50
5·14
4·12
3·36
3·16
2·46
1·96
1·76
1·16
1·00
0·98
0·00
from ⊙C go South west
⊙D
⊙d
⊙C
To School



⊙B
8·53
8·32
7·92
7·84
27
6
7·32
89
63
7·05
63
Post Office
6·44
6·41
15
5·33
86
4·70
29
4·64
64
4·51
Tank
27
4·08
28
2
3·45
65
3·18
53
2·86
9
2·61
16
2·22
24
1·52
16
0·80
8
0·39
Road
0·16
0·01
from
⊙A
4
go east
⊙C
11
25
8·54
8·29
7·88
18
50
7·81
34
7·70
7·64
56
7·39
7·31
7·21
14
7·07
6·91
76
51
48
6·59
93
63
42
6·13
6·00
9
5·91
98
69
21
5·72
43
Rest House
29
71
26
67
5·60
92
63
5·35
9
5·02
44
Tank
48
4·79
4·68
38
4·49
26
⊙b
4·34
Police st.
4·09
93
76
3·80
61
26
3·76
To School
3·64
80
51
3·29
⊙a
2·83
2·60
34
2·34
6
2·16
24
102
80
2·00
99
1·69
1·44
17
25
1·34
34
47
1·02
0·76
11
0·67
0·25
go North west
⊙B
30
Khal.



The main lines are shown full black, the tie and other lines are shown broken. Usually these lines are not shown on a finished map. The offset lines are never drawn, not even in pencil.

ANSWERS

Exercise 1

1. (i) $p = 8\frac{13}{19}$; $r = 13\frac{6}{19}$. (ii) $b = 2\cdot88$; $r = 9\cdot5$.
(iii) $a = 44$; $r = 9\cdot5$; $b = 16$; $q = 4$.

2. $3\frac{3}{10}$ in. **3.** $2\frac{7}{16}$ in. **4.** 120 ft. **5.** (i) $9\frac{7}{17}$ ft.; (ii) $3\frac{1}{3}$ ft.

Exercise 2

1. 41 ft. **2.** 8 in. **3.** 9·9 in. **4.** a = 5 ft.; b = 8.66 ft.

5. 1·76 chains. **6.** 1 m. 4 fur. **7.** 28 rasi. **8.** 157 ft.

9. 105 m. **10.** 120 ft. **11.** 7·071 ft.; 7 ft. 1 in.

12. 7 chains 62 links. **13.** 6·062 ft. **14.** 5 ft. 9 in.

15. 5 in. **16.** 58 m.

17. Draw a right angled triangle with hypotenuse 2 in. and one side 1 in., the third side will be $\sqrt{3}$ in. Construct a right angled triangle with hypotenuse 6 in. and one side 5 in., the third side is $\sqrt{11}$ in. Construct a right angled triangle with sides 3 in. and 2 in., the hypotenuse is $\sqrt{13}$ in.

18. 12·6194 ft. **19.** Rs. 482–13–5 nearly.

20. 4·47 chains; 2·235 chains. **21.** a = 15 ft.; c = 17 ft.

22. a = 20 ft.; c = 29 ft. **23.** c = 7 ft. 1 in.; b = 3 ft.

24. c = 5 ft. 5 in.; a = 4 ft. 8 in. **25.** c = 32 yd. 1 ft.; b = 24 yd.

26. a = 1 chain 20 links; c = 2 chains 41 links.

27. a = 6 chains; c = 10 chains 90 links.

28. a = 35 ft.; b = 12 ft. **29.** a = 4 ft.; b = 1 ft. 2 in.

30. a = 6 yd. 2 ft.; b = 7 yd.

31. a = 4 chains 80 links; b = 1 chain 40 links.

32. a = 3 chains 50 links; b = 1 chain 20 links.

33. a = 4 rasi 4 katha; b = 1 rasi 15 katha.

34. 168 yd.; 126 yd. **35.** AB = 15 m.; BC = 36 m.

36. 637 ft.; 245 ft. **37.** 12637 ft.; 12012 ft.

38. 1 ft. **39.** 8 ft. **40.** 80 ft. **41.** 46·861 m.
42. 1. **43.** 44 ft. **44.** 16 ft. **45.** 40·588 ft.
46. 12·648 ft. **47.** $16\sqrt{2}$ in. **48.** 12 ft. **49.** 136 ft.
50. 36 ft. **51.** 26 m. **52.** $33\frac{1}{3}$ ft. **53.** 32 ft.
54. 25 ft. **55.** 34·64 ft.; 120 ft. **56.** 6·928m.; 8m.
57. D = 86·6 ft.; 236·6 ft.; 86·6 ft. **58.** 69·282 yd.
59. h = 75 ft.; d = 72 ft.

Exercise 3

1. (i) 13 sq. yd. 2 sq. ft. (ii) 2701 sq. ft.
(iii) 30 sq. yd. 8 sq. ft. 57 sq. in. (iv) 1·92888 acres.
(v) 17b.4k.$5\frac{3}{5}$ ch. (vi) 56b.5k.$9\frac{3}{10}$ ch.

2. (i) 25·3 acres. (ii) 148·8 acres. (iii) 2·45376 acres.
(iv) 10·00392 acres. (v) 11·6848 acres. (vi) 1·575 acres.
(vii) ·73125 acres.

3. (i) 28 sq. ft. 64 sq. in. (ii) 140·625 acres.
(iii) 3·28329 acres. (iv) 7·43044 acres.
(v) 20b.18k.$9\frac{4}{5}$ ch. (vi) 205b.7k.$11\frac{9}{80}$ ch.
(vii) 48·05 acres. (viii) 9·05858 acres.
(ix) 624 sq. ft. 32 sq. in. (x) 5·42882 acres.

4. (i) 3 yd. (ii) 60 yd. 1 ft. 6 in. (iii) 80 yd. 2 ft.
(iv) 10 chains. (v) 9 yd. 2 in. (vi) 5b.12k.8 ch.

5. (i) 1·5 chains (ii) 176 yd. (iii) 6·25 chains. (iv) 3b.

6. (i) 21 yd. 1 ft. 7 in. (ii) 23 chains 40 links.
(iii) 12 chains 40 links. (iv) 311 yd. 4·5432 in.
(v) 207 yd. 1 ft. 10·32 in.

7. (i) 6 yd. 2 ft. 8 in. (ii) 12 chains 6·59 links.
(iii) 32 chains 39·5 links. (iv) 207 yd. 2 ft. 3·648 in.
(v) 5b. 18k.

8. 7·225 acres. **9.** Rs. 2430. **10.** 1120 sq. ft.
11. £4 5*s.* $4\frac{11}{36}$*d.* **12.** Rs. 78–11–$3\frac{1}{9}$. **13.** 198 ft.
14. 1·16281 acres. **15.** Rs. 4–2–0. **16.** Rs. 760.
17. 27 ft. 4 in. **18.** Rs. 1–5–$1\frac{107}{121}$. **19.** 15 min.
20. Rs. 363. **21.** 116 yd.; 85 yd. **22.** 12 chains; 5 chains.
23. 5 min. 48·75 sec. **24.** 50 chains; 40 chains.
25. 68 yd. 1 ft. $6\frac{2}{3}$ in. **26.** 672. **27.** 25 ft.
28. Rs. 485–5–4. **29.** Rs. 235–9–$7\frac{1}{5}$. **30.** Rs. 1462–8.

31. 385 ft.
32. $40\frac{1}{2}$ in.
33. Rs. 52–12–$9\frac{3}{5}$; Rs. 278–6–$4\frac{4}{5}$.
34. 263 sq. ft. 90 sq. in.
35. 780 sq. ft.
36. 5 ft.
37. Rs. 43–9–6.
38. 12 ft.
39. 463.
40. 20 rows; 540 tiles.
41. 3 ft.
42. 2 ft. 1 in.
43. 42 ft. 8 in.
44. 11; 11.
45. 105·6 yd.; 34·4 yd.; no; least perimeter is 241 yd.
46. 1·285 too little.
47. 540 sq. ft.
48. 23880.
49. £37 16*s*. $4\frac{1}{2}$*d*.
50. $1782\frac{1}{2}$ sq. ft.
51. 70·5 poles or 387 yd. 2 ft. 3 in.
52. 24 yd.; 36 yd.
53. l. 31 ft.; b. 21 ft.; h. 13 ft.
54. 3·84 acres; 2·16 acres.
55. 8·653 ft.
56. 9 ft.
57. $9\frac{3}{4}$ in.
58. 12 ft.
59. $13\frac{1}{2}$ ft.
60. £33.
61. 18 ft.
62. 6·49 yd.
63. $40\frac{2}{9}$ sq. ft.
64. 8·866 ft.
65. 33.
66. 18·5017 ft.
67. 98 yd. 2 ft.
68. 120 ft.; 80 ft.
69. 50 min.
70. Rs. 35–7–0

Exercise 4

1. (i) 39 sq. ft. (ii) 8·95524 acres. (iii) 13b. 11k. $9\frac{3}{5}$ ch. (iv) 3 acres 0 roods 10 poles.

2. 12·54 acres.

3. (i) 17 ft. (ii) 10 chains. (iii) 101·41 yd.

4. (i) 6 ft.; 2 ft. 6 in. (ii) 176 yd.; 15 yd. 1 ft. (iii) 20·90 chains; 12 chains.

5. 125 yd.; 100 yd.; 75 yd.

6. 7·80 chains; 6·72 chains; 3·96 chains.

7. 154 yd.; 550 yd.; 528 yd.

8. 60 chains; 65 chains; 25 chains.

9. (i) 221 sq. ft. (ii) 16 sq. yd. 5 sq. ft. 72 sq. in. (iii) 144 sq. yd. 6 sq. ft. 128 sq. in. (iv) 23·251305 acres. (v) 19 b. 6 k. $1\frac{3}{5}$ ch.

10. (i) 12 ft. (ii) 23 yd. 2 ft. 4 in. (iii) 40·96 chains nearly. (iv) 48·72 chains nearly.

11. (i) 4 ft. $7\frac{5}{16}$ in. (ii) 13·44 chains. (iii) $28\frac{5}{16}$ ft.

12. (i) 140·29605 sq. ft. (ii) 14·73445 sq. yd. (iii) 6·3188 acres nearly. (iv) 3·89711 b. (v) 27·0625 acres.

13. 24·969 yd. **14.** 533 yd. 1 ft. 4·97 in.

15. 33·255 acres. **16.** 6.928 yd.

17. (i) 10 sq. yd. (ii) 306 sq. ft. (iii) 2·25 acres. (iv) 1030·6296 sq. yd. (v) 29·88216 acres.

18. 192 sq. ft. **19.** 480 sq. ft. **20.** 26 ft.

21. 25 ft. **22.** 20·7056 yd. or 77·2740 yd.

23. £3 6*s.* 8*d.* **24.** 7·888 yd. **25.** 48 ft. 1 in.

26. 28 ft. 10 in. **27.** 4·7 in.; 4·7 in.; 6·6 in.

28. 21 ft.; 20 ft. **29.** 12 yd. 1 ft.; 6 yd. 2 ft.

30. 25 chains; 12 chains. **31.** 2750 yd.; 1870 yd.

32. 17 chains. 50 links. **33.** 15 ft. **34.** 28 ft.

35. 60 ft. **36.** 22·7 in. **37.** 14·4 ft.

38. 12 ft. **39.** 84 sq. ft. **40.** 486 sq. in.

41. 2772 sq. ft. **42.** 21·217 sq. ft. **43.** 169·705.

44. 3570 sq. yd. **45.** 43·301 sq. ft. **46.** 28.9 ft.

47. 317 ft. nearly. **48.** 39·6862 sq. ft.

49. 10 acres 1 rood 33 perches nearly.

50. $4537\frac{1}{2}$ sq. yd.; $2722\frac{1}{2}$ sq. yd.

51. 3 acres 2 roods 9·088 poles. **52.** 72·74 ft.

53. 45 ft.; 630 sq. ft.; 540 sq. ft.

54. 192·837 sq. links. **55.** 93944·755 sq. yd.

56. 21 acres. 1 rood 12·7 perches. **57.** 233 sq. yd. 3 sq. ft.

58. 22·4 yd.; 25·84 yd.; 24 yd. **59.** 0·537 acres.

60. 1 acre 1 rood $24\frac{112}{121}$ poles. **61.** 620964 yd. nearly.

62. $119\frac{1}{21}$ sq. yd. **63.** 15 acres 0 roods 37·91 poles.

64. 1 rood 26·6 poles. **65.** 9841·251 sq. ft.

66. 3468 sq. ft. **67.** 120 sq. ft. **68.** 1 acre.

69. Rs. 407–5–0. **70.** $8\frac{1}{2}$ in. **71.** 99; 15.

72. £2 15*s.* **73.** 305·8 sq. ft; 369·6 sq. ft.

74. £12 18*s.* **75.** 30; $6\frac{1}{2}$. **76.** 14 chains 15·3 links.

77. 24 ft. **78.** 58·8 poles. **79.** 216·50 ft.

80. 56·3 sq. links.
81. 34·64 ft.; 519·6 sq. ft.
82. 4·2 yd.
83. £4.
84. 2400; 2600; 1800; 3200 sq. ft.
85. 709 ft. nearly.
86. £1218 19*s*. $3\frac{1}{5}$*d*.
87. 125·9 links.
88. 67 ft. nearly.
89. £10 4*s*. 9*d*.
90. 57·19 ft.
91. 128·9 ft.
92. £4.
93. 202 ft. nearly.
94. 240 yd.
95. 14·941 in.
96. 1·5 ft.
97. — — —
98. 1120 sq. yd.

Exercise 5

1. (i) 350 sq. ft. (ii) 95 sq. yd. 3 sq. ft.
(iii) 12·69632 acres. (iv) 4·075 acres.
(v) 121 acres. (vi) 19 b. 0 k. $12\frac{1}{5}$ ch.
(vii) 26 sq. ft. 94 sq. in. (viii) 7·72096 acres.
(ix) 182·38422 sq. ft. (x) 4·0748 acres.
(xi) 15·8744 acres.
2. 8 chains 33 links.
3. 40 ft.
4. 170 ft.
5. 4 yd. 1 ft. 3·615 in.
6. (i) 2 sq. ft. 48 sq. in.; 2 ft. 1 in.
(ii) 8·25 acres; 9·3 chains.
(iii) 14 b. 14 k. $6\frac{2}{5}$ ch.; 3 r. 18·816 k.
(iv) 108 sq. yd. 3 sq. ft.; 10 yd. 1·7056 ft.
7. 86·6 sq. in.
8. £120 2*s*. 6*d*.
9. 390 sq. yd. 1 sq. ft. nearly.
10. 1·32 chains.
11. 72 ft.; 30 ft.
12. (i) 1120 sq. ft. (ii) 27 sq. ft. (iii) 18·2412 sq. chains. (iv) 43 b.
13. (i) 10 chains. (ii) 127 ft. 6 in. (iii) 25 chains 10 links.
14. Rs. 1414.
15. 750 sq. yd.
16. 80 yd.
17. 25 chains.
18. Rs. 427–8–0.
19. 779·4 sq. ft.
20. (i) 180 sq. in. (ii) 21500 sq. ft. (iii) 150 sq. yd.
(iv) 9·72 acres. (v) 30·56806 acres.
21. Rs. 1583–9–$7\frac{1}{5}$.
22. 17 ft.; 9 ft.
23. 17 sq. yd. 8 sq. ft. 72 sq. in.
24. 210 sq. in.
25. 29·4 acres.
26. 20.15 acres.
27. 2886 sq. ft.
28. 234 sq. ft.
29. 516 sq. ft.
30. 10·3177 acres.
31. 5 ft.; $4\frac{1}{5}$ ft.
32. 3456; 60.
33. 2 acres 47·25 cents.
34. 8000 sq. ft.
35. 8 ft.
36. 256 sq. in.; $8\sqrt{5}$ in.
37. 729·15 ft.; 329·15 ft.

38. 1200 sq. yd.; 144·2 yd.; 33·28 yd.

39. 10296 sq. ft.; 125 ft.; 82·368 ft.

40. 69·71 ft.; 627·39 sq. ft

41. 346·427712; 20·0016.

42. 1054 ft.; 625 ft.; 566·63 ft.

43. 1350 sq. ft.; $37\frac{1}{2}$ ft.; 36 ft.

44. 384 sq. ft. **45.** 2 ft. 1 in.; 2 sq. ft. 48 sq. in.

46. £120 2*s*. 7*d*. **47.** 2400 sq. ft.; 50 ft.; 48 ft.

48. 1764 sq. ft. **49.** 7 chains 70 links. **50.** 12054 sq. ft.

51. 114 sq. ft. **52.** 78 sq. ft. **53.** 7500 sq. ft.

54. 204 sq. yd. **55.** 125000 sq. ft. **56.** 77 yd.

57. 2 acres 2 roods 8 poles nearly. **58.** 210 sq. yd.

59. 31·12125 acres. **60.** 17·632 acres.

61. 48·989 nearly; 844·94 nearly.

62. 10833 sq. yd. nearly. **63.** 23 ft.; 27 ft. **64.** 216.

65. $128\frac{1}{4}$ sq. yd. **66.** $3\frac{3}{7}$ ft.; $4\frac{4}{7}$ ft.; or $\frac{4}{7}$ ft.; $\frac{16}{21}$ ft.

67. 1634·99 sq. ft. **68.** 20 acres. **69.** 36·9334 acres.

70. 45·033 sq. ft. **71.** 262·6 sq. ft. **72.** 560 sq. ft.

73. 1229·8 sq. ft. **74.** 52330·3. **75.** 9.

76. $101\frac{23}{27}$ yd. **77.** 4549·92 sq. ft. **78.** 1 ft.; 2 ft.

79. 720 sq. yd. **80.** $885\frac{1}{3}$ sq. ft. **81.** $13\frac{3}{4}$ ft.

82. 143 yd. **83.** 3120 sq. chains.

84. $\frac{a+b}{4(a-b)}\sqrt{(a-b+c+d)(c+d-a+b)(a-b-c+d)(a-b+c-d)}$

85. 540 sq. in. **86.** 600 sq. yd. **87.** 49470 sq. yd.

Exercise 6

1. 3993 sq. ft. 118 sq. in. **2.** 3·899 acres.

3. 6 chains 61 links. **4.** 509·22 sq. in.

5. Rs. 309–0–4 nearly. **6.** 23636 sq. ft. nearly.

7. 259·807 sq. ft. **8.** 12 acres 11 poles nearly.

9. 82·8 sq. ft. **10.** $4 : 3\sqrt{3} : 6$.

11. 3·25 yd. nearly. **12.** 36·3 ft.; 2270·4 sq. ft.

13. $\sqrt{2}$ ft.
14. 649500 sq. ft.
15. 2338·26 sq. ft.
16. 1931·3 sq. ft.
17. 7·0295 ft.; 119·29 sq. ft.
18. 15·00 ft.
19. 5625 sq. ft.; 6495 sq. ft.
20. 1602 sq. ft.
21. 546 sq. ft.; 13 ft.
22. 27813 sq. ft. nearly.
23. 950 sq. yd.
24. 7230 sq. yd. nearly.
25. 162463·9375.

EXERCISE 7

1. (i) 11 ft. (ii) 51 yd. 1 ft. (iii) 7 chains 92 links. (iv) 4 rasi 8 katha.
2. (i) 1 ft. 9 in. (ii) 9 yd. 1 ft. (iii) 10 yd. 2 ft. 8 in. (iv) 73 chains 88·5 links.
3. 28 in.; 30 in.
4. Rs. 51–5–4.
5. 70 yd.
6. 12 miles per hour.
7. (i) 962 sq. yd. 4 sq. ft. 72 sq. in. (ii) 1·386 acres. (iii) 2·98144 acres. (iv) 74 sq. yd. 2 sq. ft. 58 sq. in.
8. (i) 7 ft. (ii) $7\sqrt{10}$ in. (iii) 1·4 chains.
9. Rs. 41–14–6 nearly.
10. £3 11*s.* 9*d.*
11. 14 ft.
12. 2 ft. 10 in.
13. 21 links.
14. 7 chains 50 links.
15. $16\frac{16}{21}$ in.
16. 11 ft.
17. 54°.
18. 19 chains 60 links.
19. 9 chains 44 links.
20. 3 ft. $6\frac{2}{3}$ in.
21. 9 ft. or 145 ft.
22. 9° 43′ 44″.
23. $134\frac{2}{21}$ sq. in.
24. 20·8828 sq. chains.
25. 7·6752 in.
26. 210 sq. in.
27. 71° 35′$\frac{5}{11}$.
28. 11 ft.
29. 28·57142 sq. in.
30. ·57 sq. ft.
31. 6·40752 sq. chains.
32. $39\frac{2}{7}$ sq. in.
33. $61\frac{38}{105}$ sq. in.
34. 19·4917 sq. ft.
35. 574·086 sq. ft.
36. 12 ft.; 30 ft.
37. 16 yd.; $36\frac{1}{4}$ yd.
38. 235·7142 sq. in.; 942·8568 sq. in.
39. 16·568 ft.
40. 176 sq. ft.
41. 39·242 yd.
42. £833 17*s.* 3*d.*
43. 76·21 yd.; 38·105 yd.; 30·40 yd.
44. 4840 yd.
45. 10 ft.
46. 79·5772 yd.
47. $275700\frac{6}{7}$ sq. ft.
48. 3·988 yd.
49. 12727·27 sq. yd.
50. 22·50 ft.
51. 75·42 ft.
52. 105 yd.
53. 13[illegible]/7 sq. ft.
54. $3\sqrt{3} : 2\pi$.
55. 14 ft.

56. $a = b(2 \pm \sqrt{2})$. **57.** 22008·33 sq. poles. **58.** £19 0*s.* $3\frac{3}{7}$*d.*

59. £75 19*s.* 5·544*d.* **60.** 4·9463 in. **61.** 5·023 m.

62. Rs. 6600. **63.** $5854\frac{6}{11}$ sq. ft.

64. 4,320,662½ sq. ft.; 7370 ft. **65.** 21·46 sq. in.

66. Triangle = 36 sq. ft.; Circle = $103\frac{1}{11}$ sq. ft. **67.** 15·093 in.

68. 7 yd. **69.** $11764\frac{2}{7}$ sq. ft. **70.** 1·818 ft.

71. 13 ft. **72.** 12·6061 in., or 71·3938 in.

73. 100 ft.; 13·397 ft.; 51·76 ft. **74.** $4223\frac{1055}{1056}$ m.

75. 11·09 in. **76.** 26·321. **77.** 29 ft.; 22·06 ft. nearly.

78. 10 ft.; 50 ft. **79.** 5·744 ft.

80. 125·62 ft. **81.** 10 in. **82.** $6\sqrt{2}$ ft.

83. 1·5. **84.** 102 ft. **85.** 314·159 sq. ft.

86. 1003 ft. nearly. **87.** 0·1612 sq. ft. **88.** 12π sq. in.

89. 908 nearly; 30520 nearly. **90.** 48·062 yd.

91. 400 sq. ft. **92.** $0·3151a^2$ nearly (a = side of square).

93. 8326·1. **94.** 1·75 ft.; 0·278 sq. ft. nearly.

95. 1850 sq. ft. nearly. **96.** 125·05. **97.** 1·732 ft.

98. 293 ft. **99.** 20 in. **100.** 17084·8 sq. ft. nearly.

101. 63·8292 sq. chains.

Exercise 8

1. 3 in. **2.** 90 sq. in. **3.** 2 ft.; 2 ft. 2 in.

4. 16 in. = 1 mile. **5.** 70·7105 ft. **6.** 69·28 ft.; 97·97 ft.

7. 169 : 289. **8.** 42·25 sq. in. **9.** 31½ sq.ft.; 94½ sq.ft.

10. $4\sqrt{2}$ ft. **11.** 8 in. = 1 m. **12.** 1 in. = $\sqrt{10}$ chains.

13. 117 ft.; 156 ft.; 195 ft. **14.** 70 acres 85·6 cents.

15. 6 ft.; 8 ft.; 10 ft. **16.** $10\sqrt{3}$ in.; $10\sqrt{2}$ in.; 10 in.

17. 28 : 3. **18.** 8·049 in. **19.** 1116·9 sq. ft. nearly.

20. 111·8 ft. **21.** 40 ft.; 50 ft.; 20 ft.

22. 7·46 acres; 269·4 yd. **23.** $4\sqrt{5}$ ft; $4\sqrt{10}$ ft; $4\sqrt{15}$ ft.; $4\sqrt{20}$ ft.

24. 10 in.; $10\sqrt{2}$ in.; $10\sqrt{3}$ in.; 20 in. **25.** 376 ft. 2·1 in.

26. 459. **27.** 2549·1 ft. **28.** $\frac{\text{Diameter.}}{\sqrt{3}}$

29. 44·58876 in. **30.** $50\sqrt{3}$ ft.; $60\sqrt{3}$ ft.; $80\sqrt{3}$ ft.

Exercise 9

1. 44 sq. ft.
2. 56·8275 sq. ft.
3. 8·82 sq. in.
4. 2580 sq. ft.
5. 2880 sq. ft.
6. 94000 sq. ft.
7. $33093\frac{1}{3}$ sq. ft.
8. 3900 sq. ft.
9. — — — — —
10. 924·6 sq. ft.

Exercise 10

1. 32; 42; 61.
2. 81; 95; 48.
3. 899; 976; 989.
4. 1006; 2005; 4003.
5. 6844; 9876.
6. ·07; ·55; ·66.
7. 13·7; 76·1.
8. ·14001.
9. 17·01.
10. 1·3001.
11. ·9283; ·4309.
12. 1·3925; ·6463.
13. 1·5874; 1·7100; 1·8171.
14. 3·9365; 4·3268; 11·0302.
15. ·9615; ·9694; ·9737.

Exercise 11

1. (i) 512 cubic ft.; 384 sq. ft.
   (ii) 166 cubic ft. 648 cubic in.; 181 sq. ft. 72 sq. in.
   (iii) 16 cubic yd. 4 cubic ft. 163 cubic in.; 38 sq. yd. 3 sq. ft. 6 sq. in.
2. £1 12*s.* 8*d.*
3. (i) 189 cubic ft.; 201 sq. ft.
   (ii) 194 cubic ft. 168 cubic in.; 220 sq. ft. 28 sq. in.
   (iii) 7 cubic yd. 23 cubic ft. 744 cubic in.; 29 sq. yd. 2 sq. ft. 100 sq. in.
4. Rs. 70–13–4.
5. 400 gallons.
6. 25 tons.
7. 148 days.
8. 18432.
9. $4\frac{1}{2}$ in.
10. 1 ft.
11. 27.
12. 58.
13. 16 ft.; 15 ft.
14. 1 in.
15. 39·051 ft.
16. Lengthwise 932; Crosswise 995.
17. $39\frac{73}{108}$ lb.
18. 8 ft. 8 in.
19. 304·8 lb.; 663·6 lb.
20. 18 sq. ft. 108 sq. in.; 5 cubic ft. 906 cubic in. nearly.
21. 3 sq. ft. 18 sq. in.; 649·5 cubic in.
22. 21 in.; 16 in.
24. 2 ft. 6 in.
25. 16 sq. ft. 96 sq. in.
26. 60·53 in.
28. 42·4263 ft.
29. 6 days nearly.
30. 9 cubic ft.; 3182 cubic in.
31. [illegible] oz.; 1[illegible] oz.

32. 5196·15 cubic in. **33.** $17\frac{1}{2}$ ft. broad; 13 ft. high.
34. 31517 bricks. **35.** 60 in. **36.** 0·000046 in.
37. 2·75 in. nearly. **38.** 279 ft.; 93 ft.; 124 ft.
39. £1 0*s.* 8*d.* **40.** 13·61 ft.; 46·03 sq. ft.
41. 103·923 yd.; Rs. 1064 nearly.
42. ·7217 cubic ft.; 5·866 sq. ft. **43.** 62·016 oz.
44. 110 cubic in.; 1 sq. ft. **45.** 1·804 cubic ft.
46. 21·65 cubic ft.; 60 sq. ft. **47.** 1620 cubic ft.
48. 30 in.; 16 in. **49.** 22·44672 cubic in. **50.** (i) 770 cubic in.; 220 sq. in. (ii) 24 cubic ft. 108 cubic in.; 27 sq. ft. 72 sq. in. (iii) 308 cubic in.; 1 sq. ft. 32 sq. in. (iv) 43 cubic ft. 1156 cubic in.; 42 sq. ft. 112 sq. in. **51.** 14 in.
52. 1188 sq. in. **53.** 404 cubic ft. 432 cubic in.
54. 1087 coins. **55.** Rs. 56–9–$1\frac{5}{7}$. **56.** 3·928 in.
57. ·006 in. **58.** 960·3 gallons.
59. 0·163 cubic ft.; 73·6 lb.
60. 1 m. $50\frac{5}{11}$ s. **61.** 737·2 sq. in.; 1508·57 cubic in.
62. Rs. 349–3–$3\frac{13}{21}$. **63.** 62·86 cubic in. **64.** 39·13 cubic in.
65. 1609 tons. **66.** 104·7 ft.; 142·1 lbs.
67. 1 in. **68.** 220·98 cubic in. **69.** 554·26 cubic in.
70. 9·76 cubic in. **71.** ·28 in.
72. 52·8 in.; 15·4 in.; 18·2 in. **73.** $6\frac{58}{121}$ in.
74. 0·416 in. **75.** 1·8638 tons. **76.** 974·278 cubic ft.
78. $2862\frac{3}{4}$ cubic ft. **79.** $3033\frac{9}{14}$ cubic ft. **80.** 2·598 cubic ft.
81. 1848 cubic ft. **82.** $35982\frac{2}{9}$ cubic yd.; $5\frac{1}{3}$ ft.
83. 2595 cubic ft. **84.** 14627 cubic ft. nearly.
85. 2178 tons. **86.** 1697 cubic ft. nearly.
87. $153\frac{3}{14}$ cubic ft. **88.** 338 cubic ft. nearly; Rs. 118 nearly.
89. 107 ft. **90.** 3374·9 cubic ft.; Rs. 1012–7–6.
91. 2362·4 cubic ft. nearly.; Rs. 708–11–8 nearly.
92. 33750 cubic ft. **93.** 1104 cubic ft. nearly.
94. 1500 cubic ft. **95.** 157023 acres. **96.** $18327\frac{3}{11}$ ft.
97. $1643\frac{25}{112}$ cubic ft.; $670\frac{101}{112}$ cubic ft.
98. $1195\frac{1}{14}$ cubic ft.; $487\frac{13}{14}$ cubic ft. **99.** 5·9 in.

101. $1270\frac{25}{28}$ cubic ft. **102.** 373·395 lbs. **103.** 90 cubic ft.

104. 489 nearly. **105.** $1257\frac{1}{7}$ cubic ft.; 550 cubic ft.

106. 1·276 in. **107.** $97745\frac{5}{11}$ yd. **108.** $339\frac{3}{7}$ lb.

109. $0 \cdot 00\dot{5}\dot{4}$ sq. in. **110.** 5·4 gallons nearly. **111.** 1·36 ft. nearly.

112. 651·9 lb. **113.** 3·5625 gallons. **114.** $1018\frac{2}{7}$ cubic in.

115. $\frac{7}{15}$ in. **116.** 10·283 cubic in. **117.** 8200·83.

118. 3·286 cubic in.; 20·92 oz. **119.** $2\frac{1}{3}$ in.

120. $63\frac{7}{11}$ in. **121.** Rs. 5091–6–$10\frac{2}{7}$. **122.** $127285\frac{5}{7}$ cubic ft.

123. 3·841 in. **124.** 14520 tanks. **125.** 96 mins.

126. 4602 lb. nearly. **127.** 51606 galls. nearly.

128. $15639\frac{3}{11}$ yd. **129.** $15\frac{219}{784}$ cubic in.; $19\frac{311}{3136}$ cubic in.

130. $579\frac{15}{56}$ sq. ft. **131.** 1 in. or 2 in. **132.** 4.

133. 1 : 2. **135.** 14·357 in. **136.** $19\frac{1}{11}$ in.

137. $183\frac{69}{196}$ sq. in. **138.** Rs. 800.

EXERCISE 12

1. (i) 600 cubic in. (ii) 1280 cubic ft. (iii) 243·99 cubic in.
(iv) 0·5773 cubic ft. (v) 1885 cubic in. nearly.

2. (i) 616 cubic in. (ii) 132 cubic in. (iii) 1386 cubic in.
(iv) $60\frac{1}{2}$ cubic ft. (v) 1232 cubic in.

3. 114 lb. **4.** 1 ft. 6 in. **5.** $1178\frac{4}{7}$ cubic in.

6. 500 cubic in. **7.** 20 in. **8.** 1 ft.; 3 ft. 1 in.

9. $42\frac{7}{9}$ gallons. **10.** 16·8132 in. **11.** 8 sq. ft. 128 sq. in.

12. 619·8 sq. in. **13.** £3 3*s.* 9*d.*

14. 11·31 in.; 443·392 sq. in.

15. 15·65 in.; 831·55 sq. in.

16. 140·292 sq. in.

17. (i) 7 sq. ft. 48 sq. in. (ii) $35\frac{5}{14}$ sq. ft.
(iii) 14 sq. ft. $92\frac{6}{7}$ sq. in. (iv) 8 sq. ft. $7\frac{5}{7}$ sq. in.
(v) 110 sq. ft. 132 sq. in. (vi) 4 sq. ft. 128 sq. in.

18. 7 in. **19.** 7 in. **20.** 1914 sq. in.

21. 16 min. 40s. **22.** 5 sq. ft. 96 sq. in. **23.** 148 cubic in.

24. 121·16 cubic in. **25.** 122·2 sq. in. **26.** 329·09 cubic in.

27. 59·5486 cubic ft. **28.** 1350·15 cubic in.

29. (i) 616 cubic in. (ii) 2 cubic ft. 702 cubic in.

30. 9 sq. ft. 134 sq. in. **31.** 88 cubic in. **32.** 7 sq. ft. $13\frac{1}{7}$ sq. in.

33. 10*s*. $8\frac{1}{3}$*d*. **34.** 4 cubic ft. 1712 cubic in.

35. 3645·714 sq. in. **36.** 377·14 sq. in. **37.** 1575 gallons.

38. 1 cubic ft. 472 cubic in. **39.** 1·6 in.

40. 22·66 sq. yd. **41.** 19·897 cubic ft. **42.** 174 tons.

43. 0·717. **44.** 1936 cubic ft.

45. $693\frac{1}{3}$ cubic ft.; 360 sq. ft. **46.** 12 in.; 4 in.

47. 935·307 cubic ft. **48.** 81·1898 cubic ft. **49.** 1173·4 cubic ft.

50. 1885·618 cubic ft. **51.** 36373 cubic ft.

52. 203·6467 cubic in. **53.** 311·769 cubic ft. **54.** 6363·96 cubic ft.

55. 3·399 cubic ft.

56. $1493\frac{1}{18}$ cubic ft.; 801·249 tons; £104696606 8*s*.

57. $64\frac{1}{6}$ cubic ft. **58.** 391·93 cubic ft. **59.** $116\frac{27}{154}$ cubic ft.

60. $37\frac{5}{7}$ cubic in. **61.** 6268 galls. nearly. **62.** $70\frac{5}{7}$ cubic ft.

63. $288000\sqrt{2}$ cubic ft. **64.** $339\frac{3}{7}$ cubic in.

65. $18\frac{3}{4}$ ft.; 19·40229 ft.; 10·0904 ft. **66.** 63·39 cubic in.

67. 15·705 gallons. **68.** £10 10*s*. **69.** 90 sq. ft.

70. Rs. 25–8–4. **71.** 173·20508 sq. ft. **72.** £878653 nearly.

73. 17·748 sq. ft. **74.** 6·5 sq. in. **75.** Rs. 160 nearly.

76. Rs. 85–11–2. **77.** 125·6 sq. ft. **78.** 435·099 sq. ft.

79. 1·113 sq. ft. **80.** 8·2915 in.; 237·1 sq. in.

81. 236·28 sq. ft. nearly. **82.** $75\frac{3}{7}$ sq. in. **83.** 221·269 sq. in.

84. 12·03 cubic ft. **85.** £12. **86.** £83 9*s*. 7·714*d*.

87. $83\frac{17}{21}$ ft. **88.** $39\frac{1}{9}$ sq. yd. nearly. **89.** 7600 cubic ft.

90. 648 cubic ft. nearly. **91.** 147 cubic ft. **92.** 164·638 cubic ft.

93. Rs. 125077–5–4. **94.** $912\sqrt{3}$ cubic ft. **95.** 563·31 cubic in.

96. 57·62 gallons. **97.** $168\frac{14}{27}$ cubic yd.

98. 12370 cubic in. nearly.

99. 5 lb. 3·9 oz. avoir. nearly. **100.** $7372\frac{5}{14}$ cubic ft.

101. 636 times nearly. **102.** 2·05 in.

103. 1 m. 1005 yd. 1 ft. $3\frac{3}{8}$ in. **104.** 4826·25 cubic ft.

105. $917\frac{4}{7}$ cubic ft. **106.** 0·806 in. per hour. **107.** £30 2*s*. 2*d*. nearly.

108. 14·9 in. nearly; 24·9 in. nearly. **109.** 91 gallons nearly.

110. Rs. 958–9–1$\frac{3}{7}$. **111.** Rs. 14–8–6$\frac{6}{7}$. **112.** 9654 cubic ft.

113. 141·8 cubic yd. nearly. **114.** 94 lb.

115. 48 ft. **116.** 4$\frac{5}{8}$ in. **117.** 68954$\frac{2}{7}$ cubic ft.

118. 66 times. **119.** 2·94 in. **120.** 73$\frac{3}{4}$ sq. in.

121. 1100 sq. ft. **122.** 1963·5 sq. in. nearly.

123. 346·62 sq. ft.

Exercise 13

1. 45 cubic in. **2.** 1 cubic ft.; 1536 cubic in.

3. 1 cubic ft. 414 cubic in. **4.** 185 tons.

5. 1284 sq. in. **6.** 5773·5 cubic in.; 1600 sq. in.

7. 409$\frac{1}{3}$ cubic in. **8.** Rs. 1980. **9.** 1522500 gallons.

10. 12 ft., 4 ft.; 498·8 cubic ft.; 274 sq. ft.

11. 4 cubic ft. 228 cubic in.

12. 1155 cubic in.; 462 cubic in. **13.** 11$\frac{79}{198}$ in.

14. 4000 cubic in. **15.** 59 cubic in.; 161 cubic in.; 239 cubic in.

16. 35 cubic ft. **17.** 995·92 cubic in.

18. 360 cubic in.; 364 cubic in.; 1004 cubic in. **19.** 296050·4 gallons.

20. £842 4*s*. 5$\frac{1}{3}$*d*. **21.** 101600 cubic ft. **22.** 29680 cubic in.

23. 12580$\frac{20}{27}$ cubic yd. **24.** 26516$\frac{2}{3}$ cubic yd. **25.** 12466$\frac{2}{3}$ cubic yd.

26. 9·154 tons. **27.** Prismoid, 36120 cubic in.; Wedge, 7800 cubic in.

28. 47412$\frac{4}{9}$ cubic yd. **29.** 647333$\frac{1}{3}$ cubic ft. **30.** 925929$\frac{17}{27}$ cubic ft.

31. 198$\frac{2}{3}$ cubic ft.; 126$\frac{2}{3}$ cubic ft. **32.** 202,400 cubic ft.

33. 73$\frac{1}{12}$ cubic ft. **34.** Rs. 16159. **35.** 6·9282 cubic ft.

36. 72 cubic ft. 651 cubic in. **37.** 49$\frac{2}{3}$ cubic ft.

38. 38026$\frac{2}{3}$ cubic yd.

Exercise 14

1. (i) 17 sq. ft. 16 sq. in.; 6 cubic ft. 1130$\frac{2}{3}$ cubic in.
(ii) 154 sq. ft.; 179$\frac{2}{3}$ cubic ft.
(iii) 221·76 sq. in.; 310·464 cubic in.

2. 14 ft. **3.** £2 11*s*. 4*d*.

4. 78·5 sq. ft.; 70·4 sq. ft.

5. 3 in.; 36 π cubic in. **6.** 0·676 in. **7.** 264 sq. ft.

8. 51·6 lb. **9.** 1·17 oz. **10.** 4·24 in.

11. 840. **12.** (i) 3 in. (ii) 3 ft. 6 in.

13. (i) $1437\frac{1}{3}$ cubic in. (ii) 22 cubic ft. 792 cubic in.

14. (i) 154 sq. in. (ii) $12\frac{4}{7}$ sq. ft. **15.** 1945 oz.

16. $\frac{1}{2}$ in. **17.** ·5 in. **18.** 2 ft.

19. 22·6 in. **20.** 1·1 in. **21.** $31\frac{3}{7}$ cubic ft.

22. $4\frac{1}{2}$ in. **23.** 3809523 cubic m. nearly.

24. 4·06293 in. **25.** 1·2399 lb. **26.** 2 ft. 6 in.

27. $7\frac{5}{8}$ lb. **28.** π cubic ft. **29.** 864 persons.

30. $\frac{2\pi}{3}$ cubic ft. **31.** $165\cdot\dot{8}7301\dot{5}$ lb. **32.** 7·676 in.

33. 0·72204 in. **34.** 8 π cubic ft. **35.** 8·57 in.

36. 77·78 cubic ft. **37.** 168 lb. $5\frac{1}{4}$ oz. **38.** $\frac{\pi}{3}$ cubic ft.

39. 20·08 lb. **40.** 10*s*. $2\frac{58}{63}$*d*. **41.** £95 14*s*.

42. $47\frac{41}{56}$ cubic ft. **43.** 128·47 sq. ft. **44.** 5·38516 ft.

45. 18 in. **46.** 352 sq. ft. **47.** 6 ft.

48. $2818362\frac{134}{693}$ sq. m. **49.** 1386 sq. in. **50.** £466 4*s*. nearly.

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